Which are the world's wobblier currencies? Reference exchange rates and their variation

by Roger Bowden and Jennifer Zhu*

Abstract

Measuring country exchange rates relative to a common reference basket results in a set of no-arbitrage prices, unlike trade-weighted indexes, the usual method of comparing country exchange rate histories. The reference basket is analogous to a portfolio, and its choice can be resolved by drawing on required economic interpretations or uses. We use currency reference rates to examine the historical variability of different currencies over designated cyclical bands. The temporal decompositions used are those provided by wavelet analysis, which is light on maintained assumptions about data generating processes. Some countries, notably Japan and New Zealand do exhibit a powerful but irregular medium term cycle, while others are much more stable. Implications are briefly examined for investment, hedging, monetary policy and common currency studies.

Key words: Currency volatility, reference exchange rates, reference basket, spectral utility function, wavelets.

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* The authors are both with the School of Economics and Finance, Victoria University of Wellington, P.O. Box 600 Wellington New Zealand; emails roger.bowden@vuw.ac.nz , jennifer.zhu@vuw.ac.nz . Thanks for useful feedback go to seminar participants at the NZ Treasury, in May 2006.

1. Introduction

Knowing whether currency A is more variable than currency B, or than other world currencies, is an object of concern to investors, central bankers and policy makers, and to corporations faced with purchase or location decisions. The idea that currencies might differ in this respect is simple enough, but measurement turns out to be surprisingly difficult. One has to resolve two sets of problems, the absolute exchange rate problem and the variability horizon problem – long, medium or short run.

The first issue concerns the necessity to isolate the currency values for a particular country. Exchange rates are inherently bilateral, yet we need to form a judgement about this or that currency in isolation. Is the Japanese yen inherently more variable than the US dollar or the UK pound? To answer that, we need an absolute rate for the Japanese yen, the US dollar or the UK pound, to be extracted from the bilateral exchange rates between them and possibly other currencies. But any exchange rate always has to be a relative price, the price of one thing in terms of another, so that there is really no such thing as an absolute exchange rate. The respective trade weighted index (TWI) are commonly used to resolve such issues. The difficulty is that TWI's cannot be reconstructed into bilateral exchange rates, at least without arbitrage being possible. Thus the TWI's are not transactionally consistent with one another. Alternatively one could set the required absolute country rates just as the bilateral rates against US dollar, as the international base for currency no-arbitrage. This is indeed a valid choice, but it leaves the absolute rate for the US dollar as unity and constant over time, apparently the very model of stability. Or one could choose the country with the minimum rate of inflation and use its currency as the base. But a moment's reflection will show that low inflation is often the product of monetary policies that incidentally create rather unstable currencies; New Zealand is a case in point.

No-arbitrage absolute rates, in the desired sense, can be constructed by replacing a single numeraire currency with a reference basket of world currencies, and measuring each bilateral exchange rate relative to this basket. In constructing such currency reference rates (CRR's) there always remains one degree of freedom, which effectively reduces to the choice of reference base. All no-arbitrage reference rates can be constructed in this way. There is considerable freedom in the choice of reference bases, indeed some of the weights be negative as well as positive, and there are useful analogies with portfolio analysis. The reference base construction needs to be resolved in an economically meaningful way, depending on the context and use. We suggest several alternatives, each with its own shade of meaning.

The first is based on a simple geometric average of the bilateral rates, which turns out to have a minimum norm property, namely that it minimises the distance between country rates. A reference basket of this sort can be viewed as analogous to the use of a minimum variance portfolio of currencies as benchmark. It is probably the simplest way to approach arguments about whether this or that currency has been more variable than another; construct an artificial currency as stable as possible, then see if the subject currency is more variable. The centred rate, as we call it, is very easy to compute, and it can conveniently serve as a point of departure for any other alternative base rate system.

A second alternative is to bias the reference basket in one way or another, notably to conform with the trade or capital flows of a particular country of concern. The reference basket has to remain common, to avoid arbitrage. But the interpretation correspondingly changes. It is less concerned with issues as to which is the most variable currency and becomes more particular to the concerns of a specific country of choice. The apparent rates, as they are now called, are seen through the eyes of residents of the chosen country. Suppose other countries had exactly the same trade or capital flow patterns as us: would their currencies be as stable or unstable as ours relative to that reference base? This is a possible window into common currency studies. In terms of the illustration, if New Zealand had been operating off the Australian dollar, but unchanged trade weights, would it have enjoyed or more or a less volatile exchange rate history?

Other alternatives are to base spot reference rate baskets on a single set of global trade or capital weights, in effect some weighted average of each country's own trade or capital weights. An alternative starting point is to base the reference basket not on nominal exchange rates, but real exchange rates or on forward exchange rate premiums.

A second set of problems concern the measurement of variation. This sort of issue is hardly a problem if one believes that exchange rates follow a random walk, with or without drift. But (as our methodology confirms), exchange rates over longer time intervals have not, on the historical evidence, followed random walks. Some of the country reference rates exhibit a quite pronounced cyclical character, even if it is not the regularity demanded by standard (Fourier) spectral analysis. But cyclical or not, is there a sense in which one can say that this or that absolute rate is more variable over a certain horizon: the 5-6 year band of the business cycle, or the 1-2 year band or shorter? We answer this and related questions by using wavelet analysis. This is a case of new wine in old bottles. Originally called wave packets in quantum physics, the effective empirical implementation for general use is much more recent. Wavelets cater for cycles of irregular form or amplitude, as well as a more natural treatment of the trend-cycle dichotomy. Wavelet analysis is easy on maintained assumptions about linearity or nonlinearity, stationarity or nonstationarity, or as to implicit structural theorising. The orthogonal decompositions that result enable a very simple approach to the issue of variation over different horizons. Using it we can show that countries have differed in their absolute exchange rate behaviour and that some have had more pronounced variation than others, especially in certain cyclical bands. The latter finding has importance for risk management and economic policy. Hedging is easy for short run fluctuations but not so easy for the longer run, and the problem is more serious when high amplitude cycles are present, but of irregular periodicity.

It should be stressed at the outset that currency variability is not an unequivocally bad thing. A variable currency can smooth trade and other shocks, or can represent a needed adjustment to structural changes. A finding that a country's currency has exhibited a lot of variability is one that needs to be examined on its own merits. Issues of this kind are also briefly canvassed in the course of the present paper.

The scheme of the paper is as follows. Section 2 develops the reference currency rate normalisations and their interpretation on a general level. Section 3 exposits the essentials of wavelet analysis, with more detailed or technical material relegated to an appendix. Section 4 applies these methods to a set of world currencies, in both nominal and real terms, and with alternative normalisations, interpreting the results. Section 5 concludes, setting the results in the context of debates about risk management, investment, economic policy and common currency debates.

2. Construction and use of reference currency rates

As earlier noted, use of trade-weighted indexes is a common way of isolating a country's specific or intrinsic exchange rate. However trade or capital-weighted indexes have two problems: First, they refer to only a subset of foreign exchange activities. For some countries, trade flows account for a minor proportion of transactions, the bulk of the trade being driven by capital transactions. More importantly, trade weighted indices have no exchange content. Thus if A_i^{TWI} is the TWI index for country *i*, and A_j^{TWI} that for country

j, one might think of constructing a bilateral exchange rate as $R_{ij}^{TWI} = A_i^{TWI} / A_j^{TWI}$. But this will differ from the actual traded rates R_{ij} by a log linear function of the difference between the TWI weights of the two countries. Any trader who tried to price actual bilateral rates against TWI rates would not last very long. One would also want variation in currencies to take full account of their pricing interdependence – only actual traded prices do this properly.

A little structure is useful at this point; what follows is informal in nature with more explicit proofs given in Appendix A. Let R_{ij} be a bilateral exchange rate with currency i as commodity currency and country j as terms currency, so that 1 country i unit is worth R_{ij} units of currency. Write $s_{ij} = \log R_{ij}$ and let $S = ((s_{ij})); i, j = 1,...n$ be the matrix of bilateral log exchange rates. Ignoring bid ask spreads, no-arbitrage will ensure the existence of a set of country-specific prices *{A_i}*, or currency reference rates, such that $R_{ij} = \dfrac{A_i}{A_j}; s_{ij} = a_i - a_j$, where the a's are the logs. For n currencies there are only *(n-1)* independent reference rates[1], so there is one degree of freedom in choosing them.

A simple way of constructing a set of mutually consistent currency reference rates (CRR's) is to take the bilateral rates with respect to a chosen numeraire currency, notably the US dollar. This is how the market avoids arbitrages in practice. If the US is country n, we would be taking the n'th column of S, as the vector of CRR's, namely $\mathbf{a} = \mathbf{s}_n$**.** Such a choice has the advantage that adding another country to the set will not disturb the existing CRR's for the other countries. But it is not convenient in other respects, one of which is that the numeraire country automatically has zero for its absolute (log) exchange rate .One cannot easily compare the variation of currency n with that of the others.

Alternatively, we could choose for the currency reference rates any weighted combination of the columns of S, of the form

$$\mathbf{a}^w = \sum_{j=1}^{n} w_j \mathbf{s}_j ; \sum_j w_j = 1 . \tag{2.1}$$

Choosing currency n (e.g. the US dollar) as base would amount to setting $\mathbf{a} = \mathbf{s}_n$, which in turn is equivalent to setting $\mathbf{w} = \mathbf{e}_n$ the nth column of the identity matrix. A more general choice is

$$\mathbf{w} = w_1\mathbf{e}_1 + w_2\mathbf{e}_2 + ... + w_n\mathbf{e}_n \; ; \; \sum_j w_j = 1. \tag{2.2}$$

One is now replacing a single currency as the numeraire with a basket of world currencies with weights given by the vector $\boldsymbol{w}$ .We could call this a 'reference basket' or 'reference basis' for the system. The CRR a_i^w for country *i* is the bilateral rate for that currency with respect to the reference basket, viewed as though it was a currency in its own right.

In fact, all no-arbitrage CRR vectors can be constructed in this way, i.e. as some weighted average of the bilateral rates as in expression (2.1). There is no particular need to have all weights w_i semipositive – it is quite possible to think of a reference basis that is short in some currencies and long in others, rather like a portfolio, with which problem it has some features in common[2]. Because the matrix *S* of log bilateral rates has to be skew symmetric $\mathbf{w}'S\mathbf{w} = 0$, from which it follows that $\sum_j a_j^w w_j = 0$. This looks rather like a system balance of payments condition where the commodities being bought or sold in quantities w_j are the currencies at their respective prices a_j . In fact, it is just a requirement that the bilateral rate of the reference base against itself has to be zero.

The problem then boils down to the best choice of the reference basket $\boldsymbol{w}$ for the intended purpose. Some leading candidates are canvassed in what follows.

2.1 Centred rates

A simple choice is to set $w_j = \frac{1}{n}$, all j. The reference basket is a simple average of the world currencies, so that no single country is singled out for special weight. The resulting CRR vector is given by

$$\mathbf{a}^0 = \frac{1}{n}S\mathbf{1}, \tag{2.3}$$

where **1** denotes the unit vector (all elements =1). This is the simple average of the bilateral rates. The intuition is that taking an equally weighted basket creates a stable portfolio of currencies, so one might as well measure the variation of the individual currencies relative to a stable base. One can call the resulting CRR's the 'centred rates', for if $\boldsymbol{a}$ is any other CRR vector, then $a_i = a_i^0 + \overline{a}$, which means that any alternative CRR can always be represented as the centred CRR plus a common factor that adjusts for the mean.

The centred rates $\boldsymbol{a}^0$ have a minimum Euclidean norm property: among all qualifying CRR's, they minimise the length $\mathbf{a}'\mathbf{a}$. This is useful if one wants to be conservative in judgements that currencies exhibit divergent behaviour – make sure that they are as similar as possible to begin with. The minimum norm property extends to the covariance matrix Ω_a of the CRR's, assuming it exists: all other choices $\boldsymbol{a}$ have a higher trace norm than does the centred version $\boldsymbol{a}^0$ (see the remark in Appendix A). Thus the centred version can be regarded as minimising the average variation in the system.

The centred version is also useful as a point of departure for other possible reference bases. Indeed one can write

$$\mathbf{a} = \mathbf{a}^0 - (\mathbf{w}'\mathbf{a}^0)\mathbf{1}. \tag{2.4}$$

The term $\mathbf{w}'\mathbf{a}^0$ adjusts to the new base weights $\boldsymbol{w}$. The two CRR's will differ to the extent that the value of the basket $\boldsymbol{w}$ at the centred CRR's is non zero. Actually, an expression of the general form (2.4) applies to conversion between any CRR systems. For instance one could replace $\boldsymbol{a}^0$ on the right hand side with the CRR's based on the US dollar as the reference basis; formula (2.5) below is of this variety.

Country reference rate rankings will remain the same whatever the normalisation, for as expression (2.4) shows, all such normalisations differ by a constant at any particular time point. But measures of variation over time will be affected, as they will depend on the correlation over time between the centred rates and the common factor, i.e. between the two right hand terms of (2.4).

The centred rates can be computed in a very simple way. Start with the bilateral rates against any numeraire such as the US dollar, and then correct them by subtracting the average:

$$a_i^0 = s_{in} - \bar{s}_n. \tag{2.5}$$

Note that the absolute log exchange rate of the US dollar numeraire is $a_n^0 = -\bar{s}_n$ which is no longer always zero.

2.2 Apparent or myopic rates

Another approach is to look at things from the point of view of a particular country, biasing the reference basket to suit a country of primary interest. Two possible choices are:

(a) Set the reference basket for the whole system as the TWI or capital flow weights for that particular country. Note that in order to prevent arbitrage, the same weights then have to be applied to every country in the study, no matter that their TWI weights might differ from those of the country in question.

(b) Assign 0-1 type weights that give equal weight to countries that have significant trade and capital links with the country in question, but zero to others.

Comparisons based on (a) or (b) look at things very much through the eyes of one chosen nationality, hence the 'myopic' tag that we will sometimes use. The interpretation might run along the following lines: How variable is a country's exchange rate relative to others weighted in exactly the same way, just as though they all had the same TWI or capital flow weights? Suppose we applied New Zealand TWI weights to derive a CRR for Australia and found that the NZ CRR was much more variable. This does not mean that the NZD can necessarily be labelled a more variable currency than the AUD. But it might be interpreted to mean that had NZ been a (small) part of an Australian currency bloc, the conversion rate for its external trade would have exhibited less variation than it did, a topic of interest for common currency studies. A further application arises in studying inter-country differences. Suppose we choose the weights $\boldsymbol{w}$ to correspond to country A's trade or capital flows. We find that country B's CRR measured according to the same $\boldsymbol{w}$ diverges strongly from that of A over time, but that country C's CRR does not. Such a finding suggests that the country C is more similar to A than is B, with respect to its trade and capital flows. This provides a further possible window into common currency studies, this time as to the optimum selection of partners.

The precise relationship between apparent (myopic) and centred CRR's is given collectively by expression (2.4) above. The scalar $\mathbf{w}'\mathbf{a}^0$ is proportional to the angle between the vector of weights and the vector of centred CRR's; loosely, the similarity or correspondence between the two vectors. Suppose, for instance, that the home country has strong trade or capital ties with strong currency countries. Then the myopic CRR will be lower than the centred rate. The exchange rate convention says that the home country is taken as the commodity currency. The above situation will then favour home country exporters. In general, the apparent CRR may give a better indication of income or costs to local producers who export or import from abroad.

A less myopic view is a compromise reference basis for the system based on all the country TWI's. Thus if country i's TWI weights are denoted by $\mathbf{w}^j$ then the consensus based reference basket is defined by

$$\mathbf{w} = \sum_j \gamma_j \mathbf{w}^j \,, \tag{2.6}$$

where the influence or importance weights γ_j would be assigned by mutual agreement or with some particular purpose in mind. If the γ_j were chosen to coincide with country expenditures on international trade, then the result for **w** would be just the global trade weighted index. Reference baskets of the form (2.6) can be regarded as minimising a weighted sum of squared error loss functions to each country from having to conform to a common global basket (Remark 2, Appendix A).

2.3 Real absolute exchange rates

Let **P** be a vector of country consumer or producer price indices and ***p*** be corresponding vector of logs. All indices are taken with respect to a common base and time point. We define a matrix of bilateral real exchange rates as $Q = ((q_{ij}))$ with $q_{ij} = s_{ij} + p_i - p_j$

This form is often called the 'absolute version' of the log real exchange rate. The price levels take the form of consumer or producer price indices, measured off a common base year. Its changes (the dynamic version) adjust the changes in the exchange rate for differences in inflation rates.

The vector of (log) CRR's is defined by $\boldsymbol{\alpha}$ such that $q_{ij} = \alpha_i - \alpha_j$. Appendix A shows that one can construct such a vector by simply adding the difference $p_i - \overline{p}$ to any chosen version of the nominal rate:

$$\alpha_i = a_i + p_i - \overline{p} \,. \tag{2.7}$$

Use of the real exchange rates can be used to provide an alternative reference basket for the spot rates: the weights ***w*** are now chosen such that the weighted average $\mathbf{w}'\boldsymbol{\alpha} = 0$.

2.4 Forward rates

The log forward rates over the unit time horizon are defined bilaterally by

$$f_{ij} = s_{ij} + r_j - r_i \,,$$

where r_i stands for the log of the relevant interest rate factor for country *i*, or in continuous time just the interest rate itself. Collectively, the matrices of bilateral spot and forward rates are connected by

$$F = S - D; \quad D = \mathbf{r1}' - \mathbf{1r}',$$

with D as the matrix of bilateral interest rate differentials. Let ***w*** be any vector of reference basket weights. The corresponding country reference rate vectors $\mathbf{g} = F\mathbf{w}, \mathbf{a} = S\mathbf{w}, \boldsymbol{\delta} = D\mathbf{w}$ are connected by

$$\mathbf{g} = \mathbf{a} - \boldsymbol{\delta}\,. \tag{2.8}$$

Thus $\delta_i = r_i - \sum_j w_j r_j$ the gap between country i's interest rate and the weighted average.

Note that if reference baskets were chosen for the spot and forward rates independently of one another, then (2.8) would not necessarily hold.

One could choose the common reference baskets based on any of the spot rate, the forward rates or the interest rates. For instance, the reference weights could be based on those of a global cash or bond index according to the desired forward horizon (e.g. JP Morgan, MSCI indices). The property $\mathbf{w}'\mathbf{d} = 0$ would amount to specifying that the corresponding weighted average of the forward discounts or premiums is zero. In an uncovered interest parity world, this would amount to normalising the system so that the expected change in the reference basket is zero.

3. Measuring variation: exchange rate wavelets

If it can reasonably be supposed that log exchange rates obey a random walk, variational measures are straightforward: one can use the variance of log changes and leave it at that. But even in the most complete and informationally perfect international capital market, exchange rates should not even theoretically follow a simple random walk, for they depend on the cost of carry of one currency versus another, so that interest rate differentials play a role. It is stretching things to expect the latter to follow random walks. In addition, when one introduces risk into the story, risk premiums may very well not adhere to a random walk over time. There is a considerable amount of evidence that exchange rates, whether or not augmented by risk premiums do not follow random walks, especially over longer time intervals (Hodrick 1987, Engel and Hamilton 1990, Kaminsky 1993, Levich 2001, Guo and Savickas 2005). The methodology of the present paper reinforces such findings; the patterns are not those of a random walk, with or without drift, though it does not make any conjectures as to whether this is consistent with rational expectations or the market risk premium story. Beyond such issues is the further one concerning the extent to which, in making empirical judgements, it is legitimate to rely on empirical models that have embedded structural assumptions. Time series models such as ARIMA or VAR and nonstationarity specification testing are open to such objections, with assumptions as to linearity or the choice of input variables to drive the data generating processes (DGP). The methodology used in the present study, namely wavelets, makes most sense when the underlying DGP is nonlinear, but it encompasses linearity quite satisfactorily as well.

In what follows, we draw on methodology that is light on maintained assumptions, yet powerful in its ability to decompose temporal variation into horizons: the long, medium, or short run, or finer gradations. Though of much older genesis (wavelets are called ‘wave packets’ in quantum physics), empirical wavelet analysis developed rapidly following a burst of activity in the early ‘nineties directed at new representations and computational techniques by authors such as Mallat (1989), Daubechies (1988, 1990, 1992), Coifman *et al* (1990), Cohen *et al* (1992). For useful reviews of the use of wavelet analysis in economics, see Ramsay (1999), Schleicher (2002), or Crowley (2005).

A wavelet is rather like a sinusoid localised at a particular point in time, so that its power drops off rapidly on either side of that time point (see Appendix B for pictures). Wavelets come in with different scales. Thus one might have a scale representing a 6-12 month fluctuation (loosely, cycle), another for a 1-2 year, a third for 3-5 year and so on. Moving along through time, one fits a succession of wavelets for each scale. Each time point contains contributions from wavelets of the same ‘scale’ (quasi frequency) but centred at neighbouring points. This feature enables one to model cycles that do not remain constant in amplitude, so that in this respect, wavelet analysis overcomes the limitations of ordinary spectral analysis – the resulting fit might look nothing like a regular sine wave (see some of the figures below). The results of fitting a given scale are called ‘details’. It is the details associated with different scales that constitute the short, medium, long (etc) fluctuations in the given series. The variations can be measured as the average variance (called in this context the ‘energy’) at each level of detail. Thus country A might have much higher energy than country B in the 5-6 year detail (i.e. time band or period), but a lower energy in the very short detail.

In addition, each time point will also contain contributions from wavelets of different scales, corresponding to cycles of different frequencies. By a similar mathematical argument to complex demodulation, one can express the series at any point in time as a sum of the wavelets of different scales. The shorter scales represent higher frequency fluctuations, while the large scale wavelets capture the long run movements. A more detailed account is given in Appendix B, which also depicts the wavelet family used in the present study, namely the Coif 5 wavelets. Collectively across different scales, the wavelets of either family are flexible enough to allow for asymmetric local cycles of rather arbitrary form, so this is no longer a story requiring regular sinusoidal patterns.

Although all chosen from the same generic family, the wavelets are normalised to refer either to the cycles (‘mother wavelets’) or long term trend or quasi trends (‘father wavelets’). The results of fitting mother (cyclical) wavelets of different scales are the details (*D*) and they are additive in their effect. Progressive sums, by adding more details, are called the ‘approximations’ (*A*). Figure 1 is a schematic decomposition of this multi resolution analysis. Level 1 is the smallest scale or highest quasi frequency, so D_1 represents the cycle at this highest level of detail. The given series is then split into D_1 + A_1, where A_1 is the series once the very shortest fluctuations have been removed. Levels 2,3…. contain successively less small-scale complexity. Extracting these leads to broader time frame approximations designed to reveal longer run cycles and ultimately the trend. An ‘average period’ construct for a given level of detail *D* can be derived by finding the sinusoid whose period most closely matches that of the wavelet fitted at any point in time, suitably adjusted for its scale. Then one simply averages out these local equivalent periods over time. This enables us to think of the successive details as corresponding to progressively longer cycles, just as in spectral analysis.

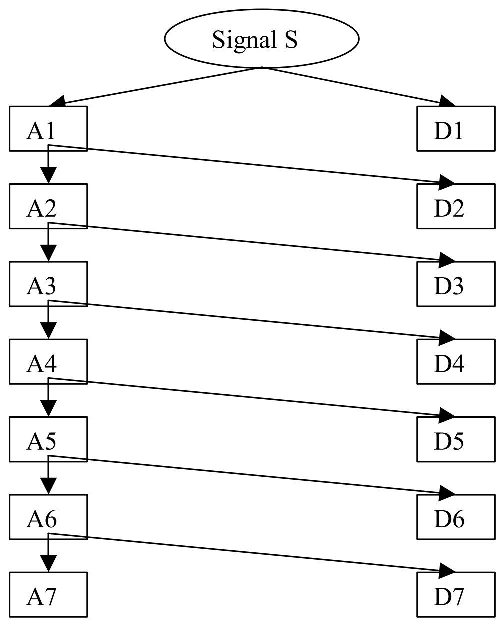

Figure 1: Decomposition into successive details and approximations

Figure 2 below illustrates with a wavelet decomposition for the NZ dollar currency reference rate using the centred version. The scale of the vertical axis of each individual graph can be taken as a rough indication of the energy at each band. A rather more precise idea can be obtained by reading off the energy decomposition in table 1. The maximum number of scales is limited by the available data. The maximum scale recognizable is of order 2^k ; thus scale 7 would require 128 months but scale 8 would need 256 months. In this case with 228 monthly observations, one can recognise cycles up to about 15 years.

The numbers in table 1 are energies, which are essentially sums of squares, i.e. have a dimension of $N \times$ variance, where N is the sample size. The detail series have zero mean by construction, but we have modified the usual definition of approximation energies(the A7 component) to centre around the mean as well. The best way to think of these detail energies is in terms of amplitudes of the relevant fluctuations. For example, the stated energy of 0.9541 for D6 translates to a notional standard deviation of $\sqrt{0.9541/228}$ =0.0649. Because these are log CRR's, this corresponds to a variation of 6.49%. One could say that if all the different observations of the CRR were projected on to a stationary distribution, the bulk of the observations would lie in a distance of twice the STD from the mean, i.e. ± 13%, which can be treated as an effective amplitude. The neighbouring D5 detail with energy of 0.1114 corresponds to a standard deviation of 2.21%, so the amplitude of the 7-8 year cycle is about three times the amplitude of the 3 year cycle.

Note in particular the accentuated peak detail energy at 6-7 years – about 73 % of all detail (cyclical) energy is concentrated here. The NZ dollar is heavily cyclical, even if an ordinary spectral analysis (not reported here) fails to support the idea that this is a very regular cycle. The combination of amplitude and irregularity makes life very difficult for corporate risk managers. The need to hedge exposures of this magnitude but to that sort of horizon is not easy to accomplish. Very short run fluctuations, on the other hand are quite easy to smooth out with market derivatives. Thus the welfare impacts of different cyclical bands are not equal and some sort of impact penalty weighting scheme should be attached. Bowden (1977) developed a similar idea in the context of traditional (Fourier) spectral analysis.

Note finally that if the series followed a random walk with a constant positive drift, one would find a wavelet decomposition with almost all of the energy in A7 and no sign of any internal peak for the details. If the drift were zero, one would find that all the energy switches to the shortest run detail. The tables in section 4 contain energy decompositions for all the other currencies of the study, both in nominal and real terms.

	Period centred at (years)	Energies	Detail energy as % all detail energy
A7	long term	0.9881	
D7	15.5	0.0814	6.2%
D6	7.7	0.9541	72.7%
D5	3.9	0.1114	8.5%
D4	1.9	0.0758	5.8%
D3	1.0	0.0311	2.4%
D2	0.5	0.0336	2.6%
D1	0.2	0.0247	1.9%

Table 1: Energy decomposition for NZ: centred nominal CRR

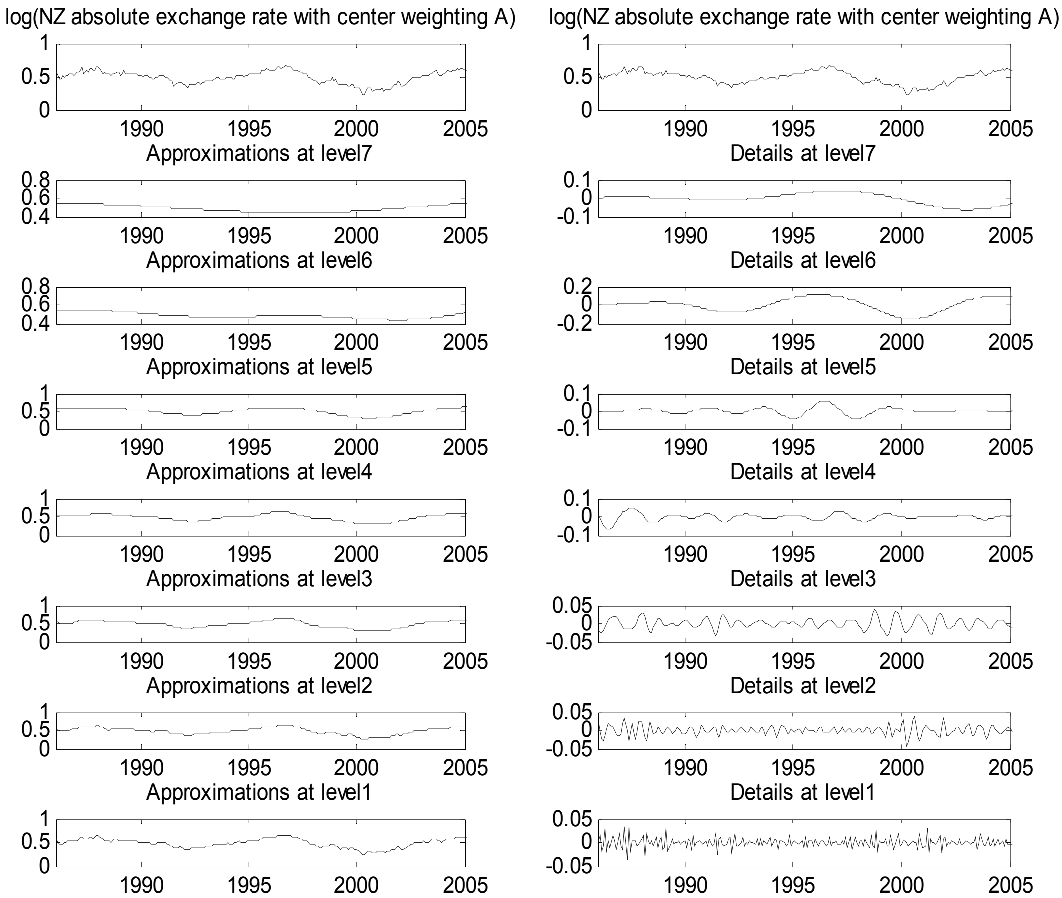

Figure 2: Details and approximations for centred NZ reference rate

4. Comparative results

The choice of currencies for the present study was influenced by the following considerations:

(a) A reasonably free exchange rate between 1986 and 2006. Many central banks do try to smooth their currencies in some way. Japan is an example of a fairly tightly managed float, while New Zealand was perhaps the world's most free float, at least up to 2005. Smoothing was allowed provided the motive was judged to be stabilisation and not currency fixing.

(b) Absence of any major structural shift that might have affected the currency, especially conversion during the sample period from fixed to floating. The one exception to this was Germany. IT was desirable to include a post 1999 Euro zone currency, and the largest European economy was chosen, notwithstanding a potential impact from German reunification in the earlier part of the period.

(c) A reasonable geographical coverage. Chile is included as the most structurally stable South American currency, in spite of doubts[3] about whether the Chilean peso is a truly floating currency, while South Africa also makes the list. We limited Scandinavia to just Sweden and Norway, the latter being an oil currency and therefore different.

Calculations cover the following, based on monthly data from 02.1986 to 11.2005. Exchange rates in all cases are logs as in section II. References are to the full tables, which appear in Appendix C.

1. Centred log exchange rates for nominal (Table C1) and real exchange rates (Table C2). The reference basket is an equally weighted in all included currencies.

2. Apparent exchange rates. We chose New Zealand as the home country, influenced also by recent debates about monetary policy and common currencies. Two versions of the reference basket $\boldsymbol{w}$ were chosen as in table 2. Weights A are simple (0-1 style) selection weights based on both the trade weights and what is known about major capital flows, such as international borrowing and its ultimate sources[4]. Weights B are the TWI weights as of 2005, except that the weights for the Euro zone are transferred in their entirety to Germany.

Currency	Weights A	Weights B
NZ dollar	0	0
US dollar	0.2	0.3396
Australian dollar	0.2	0.2370
German mark	0.2	0.1790
British pound	0.2	0.1771
Japanese yen	0.2	0.0673
All other countries	0	0

Table 2: Weights used for apparent New Zealand AER's

Tables C3, C5 give the results for weighting system A (nominal and real rates), while Tables C4, C6 do the same for weights B. Findings of interest are summarised as follows.

4.1 Centred currency reference rates

For many countries, the bulk of the power (energy) is contained in the quasi trend approximation A7, simply because log exchange rates are nonstationary. or else may imbed problematic inflation rates. The Chilean peso has depreciated markedly in both nominal and real terms so this is unsurprisingly the standout on trend. Japan is also notable in this respect.

Cyclical variation generally has lower power. However cycles (in the generalised wavelet sense) are fairly substantial for some currencies. The two standouts are Japan and New Zealand which both have sharp peaks at the D6 level, which is the 6-7 year band. The UK and the US are also variable, while South Africa has a lot of variation in the shorter D5-D3 bands between 1- 4 years.

Table 3 summarises in terms of the sum of the detail energy over all cyclical bands. There is a fair bit of concordance between nominal and real rankings. On total cyclical variation, Chile was the most unstable currency, with Japan second. The most stable currencies are Singapore and Norway, but note Australia as third most stable, ahead of Switzerland. Taking into account both cyclical and total energy, the Australian dollar stands out as a comparatively stable currency. It has much lower energy than does its trans Tasman neighbour New Zealand in the long cycles and only moderate variation in the shorter bands as well. This was a bit surprising, given that the Australian dollar is usually viewed as a commodity currency. Singapore and Norway are also stable in all bands. The Canadian dollar is appreciably more stable than is its close neighbour the US dollar, though the US dollar does have a stable band at D4 which is two years or so. The foregoing remains true for real as well as nominal exchange rate variation.

	Nominal centred CRR		Real centred CRR	
Currency	Total energy	Rank	Total energy	Rank
New Zealand	1.3121	5	1.5421	4
Canada	0.7224	10	0.8346	9
Chile	2.8490	1	1.3039	5
Germany	0.7914	9	0.5245	11
Japan	1.8508	2	1.9627	2
Australia	0.6759	11	0.8544	8
South Africa	1.8114	3	2.0543	1
Sweden	0.8622	7	0.6705	10
Switzerland	0.7942	8	1.0013	7
UK	0.8670	6	1.0203	6
Singapore	0.4421	12	0.5537	12
Norway	0.4052	13	0.3959	13
US	1.3880	4	1.7742	3

Table 3: Total detail energies

4.2 Apparent currency reference rates for the NZ dollar

In the apparent or myopic approach, everything is seen through the eyes of New Zealanders (in this application). One asks whether if the other countries had exactly the same trade or capital flow pattern, their currencies would look different or similar to the NZ dollar.

The quasi trends (A7) acquire an additional interpretation as indicating the currencies sympathetic or antipathetic to the trade patterns of NZ. Thus Germany, Japan and Singapore stand revealed as incongruous with NZ, while the US, the UK and Canada are not too dissimilar. Once again, there are differences on this count as between NZ and its neighbour Australia, manifested particularly in version B, which has the NZ trade weights

Turning to the cycles, the NZ dollar stands out even more in the 6-7 year band. This is as we should expect, for it is the currency most sympathetic to its own trade weights. Japan and South Africa would continue to be unstable, were they to adopt NZ trade weights. But once again, Australia would not share NZ’s instability.

Finally, figure 3 plots the centred CRR D6 cycle for NZ and its major trade and capital flow partners, again taking Germany to represent the Euro zone. Note the different vertical axis calibration - as earlier noted, NZ and Japan have greater power in this band. Australia and NZ share roughly the same timing, in opposition to the US, while the other countries have different patterns again.

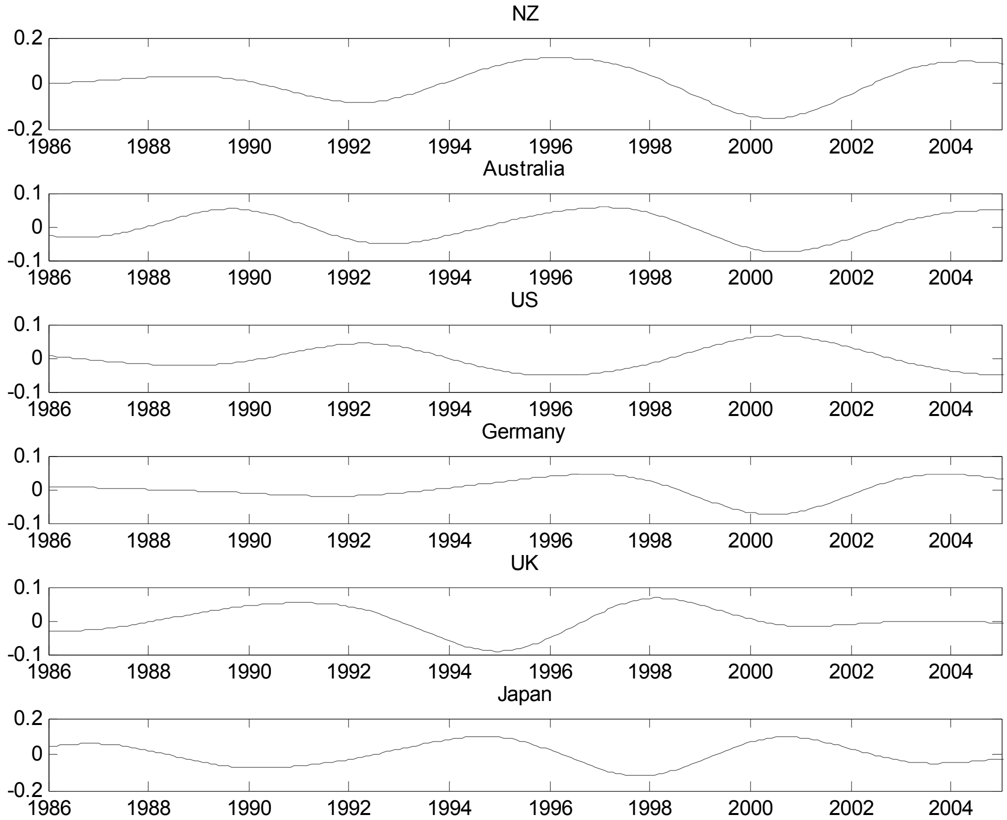

Figure 3: Detail 6 for NZ apparent weights: all included currencies

5. Further discussion and conclusions

A variation-prone currency does not necessarily mean serious welfare losses for the residents of that country. New Zealand is joint winner of the wobbliest currency award for the longer cycles. But it is also a commodity currency with a rather narrow export basis focussing on foodstuffs, the prices of which do fluctuate on world markets. To the extent that the exchange rate buffers the commodity prices and hence the economy, this is a gain and not a loss in welfare. There is some evidence of an effect of this kind up to about 1998. In such a case, welfare losses stemming from currency instability would be borne by importers or import using sectors; or by other exporters, where the prices they pay or receive are not responsive to world commodity prices. They have an inherent risk management problem with some tough decisions as to either or not to hedge the currency exposure. Hedging is not too difficult an exercise if the problem is very short run fluctuation, over a matter of months rather than years. Simple smoothing devices based on staggered forwards will suffice for this. It becomes much more difficult, with issues of passive versus active timing, if the exposures are to longer cycles, or uncertain periodicity. The existence of a forward discount on the home currency will not help its exporters if they lock into a forward rate and the home currency then devalues well beyond that level. The timing of long term forwards is not a trivial matter and it becomes more difficult if cycles are known to exist but they are irregular as to periods. In addition, long term forwards are available only to extremely good credit risks, and many exporters would not qualify, especially start up ventures. The potential risk exposure is higher the wider the amplitudes of the cycles, or in wavelet terms, the greater the energy.

Moreover, even the buffering effect on major commodity exporters is now in doubt. Bowden and Zhu (2006a) point out that post 1998, the effect has been inoperative, as the NZ economy has become increasingly exposed to flows of offshore capital. The energies may be getting larger and the problem of risk management more pressing, for all exporters and importers. It does seem to be the case that the narrower the export base, the more variable the currency. Like New Zealand, Australia tends to be regarded as a commodity currency, but this extends to oil and gas, metals and other commodity classifications which NZ does not have, and which have their own industrial usage and price cycles. But one should also note the independent influence of capital flows, particularly with the US, the UK and Japan, all of which are quite variable in their currencies yet have a well diversified export base.

An unstable currency could also be accentuated by monetary policies aimed at inflation control. A case could be made that in recent years the swings in the NZ dollar have been exacerbated by the central bank's inflation targeting regime or the way that it is run. In particular, the practice of signalling future cash rate rises has been seen as giving comfort to hedge funds and the carry trade, whereby they fund in the USD or Japanese yen and invest in NZ dollar rates. If the currency effects of their actions neutralises the commodity price buffering effect, then that is a general welfare loss to New Zealanders.

Variable currencies and their typical periodicity will also preoccupy investors and fund managers. The return on any offshore investment is the sum of its local return (or intrinsic own-currency return) and the currency return when translated back to the investor's home currency. The received doctrine is that foreign diversification is a good thing. But if currencies can exhibit pronounced cyclical variability in the currency component, uncompensated by the intrinsic return, then investors acquire an exposure to a form of timing risk. Specifically, they incur negative currency returns if their entry or exit are, for one reason or another, not well timed. This effect is more serious with the longer cycles, for the investor may not be in a position to wait for 3 or 4 years for things to improve. Bowden and Zhu (2006b) have drawn attention to this problem, proposing exit risk as one criterion for portfolio selection.

As in most forms of economic life, to become aware of a thing is to change it. If fund managers do become aware of substantial currency cycles, then they will tend to invest at the low point and divest at the high point. This, together with the ever increasing globalisation of capital markets, might mean that movements in the currency become more violent at or near peaks or troughs. There is indeed some recent evidence of this in the round of currency realignments now going on.

Appendix A: Reference exchange rates

The results and discussion that follow provides a more formal backing for the exposition of section 2 concerning currency reference rates. Proofs are mostly straightforward matrix algebra, hence only outlines are given. Some additional remarks fill out the treatment of section 2.

Nominal exchange rates

Proposition 1 (Nominal currency reference rates)

Let $S = ((s_{ij})); i, j = 1, ... n$ *be a matrix of bilateral log exchange rates with currency i as commodity currency and country j as terms currency, so that 1 country i unit =* $e^{s_{ij}} = R_{ij}$ *country j units. No arbitrage exists across currencies. Then:*

(a) A set {A_i} of CRR's exists such that $R_{ij} = \frac{A_i}{A_j}; s_{ij} = a_i - a_j$ *or collectively*

$$S = \mathbf{a1}' - \mathbf{1a}' \qquad \text{(A.1)}$$

Conversely, any bilateral set {S} defined in this way is a no- arbitrage system.

(b) Two equivalent generic representations are:

$$\mathbf{a} = \frac{1}{n}(S - \lambda I)\mathbf{1}, \qquad \text{(A.2)}$$

*where **1** denotes the unit vector (of ones) and λ is some scalar;*

$$\mathbf{a}^w = S\mathbf{w} \qquad \text{(A.3)}$$

*for some vector **w** with* $\sum_j w_j = 1$.

Representations (A.2,3) are equivalent with $\lambda = \mathbf{w}' S\mathbf{1}\,; \sum_j w_j = 1$. *Any CRR vector can be written in the form (A.2) or (A.3).*

(c) The choice λ = 0 or $\mathbf{w} = \frac{1}{n}\mathbf{1}$ *gives* $\mathbf{a^0} = \frac{1}{n}S\mathbf{1}$ *and* $\sum_i a_i^0 = 0$ *(the 'centred representation'). The centred version* $\boldsymbol{a}^0$ *corresponds to setting the log CRR for any country as the average (log) bilateral rate, with that country taken as commodity currency. For any alternative CRR vector **a**,* $a_i^0 = a_i - \overline{a}$ *and* $\mathbf{a} = \mathbf{a}^0 - (\mathbf{w}'\mathbf{a}^0)\mathbf{1}$.

(d) The centred version $\boldsymbol{a}^0$ *minimises the norm* $\mathbf{a}'\mathbf{a} = \sum_{\mathbf{i}} a_i^2$.

(e) The choice $\lambda = \mathbf{e_n}' S\mathbf{1} = \sum_i s_{in}$ *, where* $\boldsymbol{e}_n$ *is the nth identity vector, corresponds to setting* $\mathbf{a} = \mathbf{s}_n$*, in which case* $a_n = 0$*. Suppose currency n is an arbitrary base or numeraire (e.g. the US dollar) for the system of bilateral rates. The centred CRR's can be calculated as*

$$a_i^0 = s_{in} - \overline{s}_n;\ or\ \mathbf{a^0} = \mathbf{s_n} - (\frac{1}{n}\mathbf{1's_n})\mathbf{1}, \tag{A.4}$$

*(f) Given any desired weighting system **w** as in representation (A.3), the corresponding CRR's can be calculated in terms of the centred rates as* $a_i^w = a_i^0 - \sum_j w_j a_j^0$ *; or alternatively in terms of a numeraire currency as elements of* $\mathbf{a^w} = \mathbf{s_n} - (\mathbf{w's_n})\mathbf{1}$.

<u>Proof outlines</u>

(a) Existence can be confirmed by simply setting the elements of ***a*** as the bilateral rates with respect to a numeraire currency such as the US dollar. Conversely given a proposed system of reference rates, the A_i are transitive by construction, assuming they exist, and in that case we must have

$$S = \mathbf{a1'} - \mathbf{1a'}. \tag{A.5}$$

(b) Expression (1a) follows on post multiplying (A.5) by the vector **1**, rearranging and setting $\lambda = -\mathbf{1'a} = -\sum_i a_i$ .Conversely, starting from any representation of the form (1a), one obtains $\mathbf{a1'} - \mathbf{1a'} = \frac{1}{n}(S\mathbf{11'} - \mathbf{11'}S')$. The elements of the right hand matrices consist of column or row sums. But note under no arbitrage that, for example, $\sum_j s_{1j} - \sum_j s_{2j} = n(a_1 - a_2) = s_{12}$.

Turning to expression (A.3), the vector S**w** qualifies as an CRR vector. For the vector $\mathbf{a^0} = \frac{1}{n}S\mathbf{1}$ certainly qualifies as a CRR vector, so $S\mathbf{w} = (\mathbf{a}^0\mathbf{1}' - \mathbf{1}\mathbf{a}^0{}')\mathbf{w} = \mathbf{a}^0 - (\mathbf{w}'\mathbf{a}^0)\mathbf{1}$. The last vector satisfies (A.2). Conversely, given an AERV **a**, let $\lambda = -\mathbf{1}'\mathbf{a}$. Then we can certainly find a vector ***w*** such that

$$\begin{aligned} \mathbf{a}^0{}'\mathbf{w} &= \frac{\lambda}{n} \cdot \\ \mathbf{1}'\mathbf{w} &= 1 \end{aligned}$$

This would not be possible only if the elements of $\mathbf{a}^0$ were proportional to the unit vector **1**, but this cannot be the case as $\mathbf{1}'\mathbf{a}^0 = 0$. As **a** is presumed to be an CRR, expression (1a) must hold, and we can write

$$\mathbf{a} = \mathbf{a}^0 - (\mathbf{w}'\mathbf{a}^0)\mathbf{1}$$

Then $S\mathbf{w} = (\mathbf{a}^0\mathbf{1}' - \mathbf{1}\mathbf{a}^0{}')\mathbf{w} = \mathbf{a}$.

(c) and (d). The centering property follows from $\mathbf{1}'S\mathbf{1} = 0$ as S is skew symmetric. The property $a_i^0 = a_i - \overline{a}$ is a simple consequence of expression (A.2). From (A.2) and noting $S' = -S$ we obtain

$$\mathbf{a}'\mathbf{a} = \mathbf{a^0}'\mathbf{a^0} + \frac{1}{n}\lambda^2,$$

showing that $\mathbf{a}^0$ is minimum norm.

(e) If we set $\mathbf{a} = \mathbf{s}_n$ then we must have $\mathbf{e_n}'\mathbf{a} = 0$. Applying this to (A.2) and rearranging gives $\lambda = \mathbf{e_n}'S\mathbf{1} = \sum_i s_{in}$ .The calculation routine follows from part (c).

(f) See proof of part (b).

<u>Remark 1</u> (Covariance matrix)

Part (d) concerns the norm at any particular point in time. But a similar result extends over time if one is willing to assume that a covariance matrix of CRR's exists. In that case trace norm is minimised by choosing **a** = $\mathbf{a}^0$.

Proof sketch: Using part (c) we obtain

$$\Omega_a = \Omega_0 - \Omega_0\mathbf{w}\mathbf{1}' - \mathbf{1}\mathbf{w}'\Omega_0 + \mathbf{w}'\Omega_0\mathbf{w}\mathbf{1}\mathbf{1}',$$

where $\Omega_a = Cov(\mathbf{a}); \Omega_0 = Cov(\mathbf{a}^0)$ .Hence

$$tr(\Omega) = tr(\Omega_0) - 21'\Omega_0\mathbf{w} + n\mathbf{w}'\Omega_0\mathbf{w}.$$

It follows from $\mathbf{1}'\mathbf{a}^0 = 0$ that $\mathbf{1}'\Omega = \mathbf{0}'$ .Thus tr(Ω) is minimised at Ω_0.

Remark 2 (Consensus-based reference baskets)

As noted in section II, a general approach might be to seek a compromise between country TWI's or capital weighted indexes, while preserving the exchange feature. Thus let $S\mathbf{w}^j = \mathbf{a}^j$ be the CRR that might be chosen by country j to conform to its own TWI weight pattern $\mathbf{w}^j$. As noted earlier, this would have the property that $\mathbf{w}^{j}{}'\mathbf{a}^j = 0$, so if the international agreement was for a compromise weighting giving an AER vector $\mathbf{a} = S\mathbf{w}$, the loss to country j might be of the order of $\mathbf{a}'\mathbf{w}^j$. Considering all countries together, we could imagine them settling on a global weighting defined by $\mathbf{w}^g = \arg\min_{\mathbf{w}} \sum_j \gamma_j \left\|\mathbf{a}'\mathbf{w}^j\right\|^2$; $\mathbf{a} = S\mathbf{w}$ **and** $\mathbf{1}'\mathbf{w} = 1$.

In this expression, semipositive weights γ_j ; $\sum_j \gamma_j = 1$ are used to indicate degrees of importance or 'economic clout' of the economy concerned. The optimum is given by

$$\mathbf{a} = \mathbf{a}^0 - \lambda\mathbf{1} \text{ where } \lambda = \sum_j \gamma_j \lambda_j ; \lambda_j = \mathbf{w}^{j}{}'\mathbf{a}^0 .$$

Equivalently, just set the weights $\boldsymbol{w}$ as the weighted average of the individual country weights $\mathbf{w}^j$.

$$\mathbf{w} = \sum_j \gamma_j \mathbf{w}^j .$$

B: Real exchange rates

Collectively, the matrix of bilateral real rates is defined by

$$Q = S + \mathbf{p}\mathbf{1}' - \mathbf{1}\mathbf{p}'. \tag{A.6}$$

Sought is a vector $\boldsymbol{\alpha}$ of real currency reference rates such that

$$Q = \boldsymbol{\alpha}\mathbf{1}' - \mathbf{1}\boldsymbol{\alpha}' \tag{A.7}$$

Proposition 2 (Real currency reference rates)

(a) The absolute real exchange rates are generated by the form

$$\boldsymbol{\alpha} = \frac{1}{n}(Q - \lambda_{\alpha} I)\mathbf{1} ; \lambda = -\mathbf{1}'\boldsymbol{\alpha} . \tag{A.8}$$

(b) The choice $\lambda_\alpha = 0$ yields the zero centred real exchange rate $\boldsymbol{\alpha}_0$, and this is can be obtained in terms of the zero centred nominal exchange rate $\boldsymbol{a}_0$ as

$$\boldsymbol{\alpha}_0 = \mathbf{a}_0 + \mathbf{p} - \bar{p}\mathbf{1} ; \bar{p} = \frac{1}{n}\sum_i p_i . \tag{A.9}$$

(c) More generally, given any nominal exchange rate centering (value of λ), one can generate a corresponding real exchange rate as

$$\boldsymbol{\alpha} = \mathbf{a}^{\lambda} + (\mathbf{p} - \overline{p}\mathbf{1})$$

with $\lambda_{\alpha} = \lambda$.

Proof outlines

(a) As for part (a) of Proposition 1.

(b) Follows from the skew symmetry of Q, while expression (A.9) is obtained by expressing Q in terms of S using expression (A.6).

(c) Combine expressions A.6-A.8.

Remark 3

Proposition 2 gives us yet another economically meaningful value of λ for the nominal rate, namely $\lambda = \mathbf{e_n}'Q\mathbf{1} = \sum_i q_{in}$, which corresponds to setting the numeraire of the system as country n's real exchange rate.

Appendix B: Wavelet analysis

Wavelet analysis has been slow in its uptake into general empirical economics. What follows is a brief account of features of wavelet analysis as they apply to the present study.

Wavelet decompositions

Wavelets for a given family are generator functions, indexed by two parameters called the scale *(j)* and the translation or location *(k)*. For the wavelet function we employed in the paper, namely coiflets, there are two different sorts of wavelets: the father wavelet ϕ and the mother wavelet ψ. The former are normalised to integrate to unity, while the latter integrate to zero, as they are meant to span the cyclical influences. The two functions are respectively of the form:

$$\phi_{j.k}(t) = 2^{-j/2}\phi(\frac{t-2^j k}{2^j}); \qquad \psi_{j.k}(t) = 2^{-j/2}\psi(\frac{t-2^j k}{2^j}).$$

The scale parameter determines the span of the wavelet, meaning its non-zero support, as each wavelet damps down to zero on either side of its centre. For a given time *t*, there are contributions from neighbouring wavelets translated to either side of *t*. The wavelet transform based on the above function is a dyadic procedure. Therefore, the maximum level decomposition of signal cannot exceed the integral part of $\log_2^N$, where N is the number of observations.

Figures B1 depicts the two wavelet generators used in the present study.

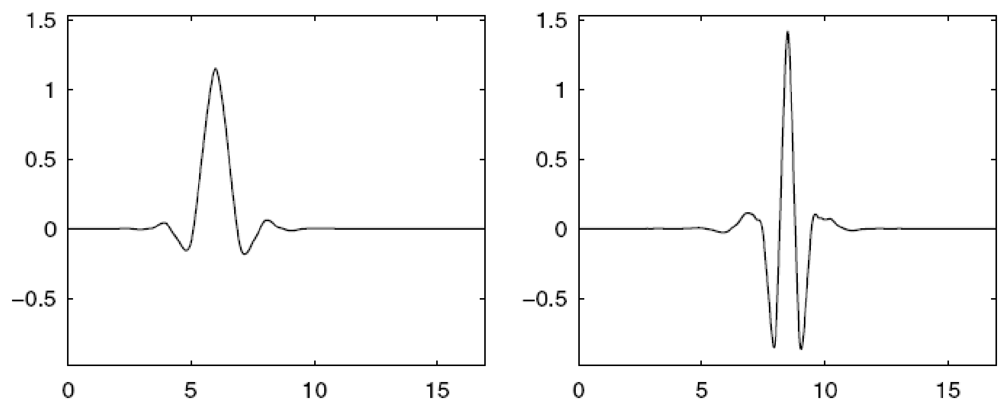

Figure B1: Coiflet father wavelet (left) and mother wavelet (right)

The family of functions defined as above are mutually orthogonal. In a manner analogous to Fourier analysis one can form coefficients as

$$s_{J,k} = \int x(t)\phi_{J,k}(t)dt\,;\;\; d_{j,k} = \int x(t)\psi_{j,k}(t)dt\,,$$

for $j = 1,2...J$, where J is limited by the number of observations available on the given series *x(t),* supposed continuous here for simplicity. As with the inverse transform in Fourier analysis, we can recover *x(t)* in terms the wavelet functions as:

$$x(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t).$$

We write $D_j(t) = \sum_k d_{j,k}\psi_{j,k}(t)$. Note that just the one father wavelet has been used in the above, with maximal scale.

Computational procedure

The quasi Fourier approach illustrated above would be slow computationally. In the present paper, computations were done in Matlab (Misiti *et al* 2005) using Mallat's algorithm, which is considerably more efficient. The algorithm follows through the basic sequence as illustrated in figure 2 of the text. The original signal *x(t)* is fed through a high pass and low pass filter, one the quadrature of the other, which ensures orthogonality of the two outputs. The low pass filter is adapted to the longer run father wavelets and the higher to the mother wavelets. Output from the high pass filter is downloaded as the level 1 detail D_1, and the output from the low pass filter becomes the level 1 Approximation. Starting afresh with A_1, the process is successively repeated.

Wavelet variances and covariances

By decomposing the time series into orthogonal components as above, the variance of components at different scales can be derived. The DWT provides a simple way of computing these that closely parallels the classical statistical formulas. For each detail level *j*, the average energy or power over the horizon can be expressed as the percentage contribution of each level of detail relative to the whole as:

$$E_j^D = \frac{1}{E}\sum_t D_{j,t}^2, \quad E_j^A = \frac{1}{E}\sum_t A_{j,t}^2$$

$$E = \sum_t A_{j,t}^2 + \sum_j \sum_t D_{j,t}^2$$

The DWT variance computations can be improved using the maximal-overlap discrete wavelet transform (MODWT) estimator of the wavelet variance (Percival 1995). We have chosen not to use this as it assumes circularity, in other words the historical series simply repeats itself.

Scale and frequency

To connect the scale to frequency, a pseudo frequency is calculated. The algorithm works by associating with the wavelet function a purely periodic signal of frequency F_c that maximizes the Fourier transform of the wavelet modulus. When the wavelet is dilated by the scaling factor 2^j, the pseudo frequency corresponding to the scale is expressed as:

$$F_s = \frac{F_c}{2^j \times \Delta},$$

where Δ is the sampling interval.

Taking the wavelet 'coif5' as an example, the centre frequency (figure B2) is 0.68966 and thus the pseudo frequency corresponding to the scale 2^5 is 0.02155. As the sampling period is one month, the period corresponding to the pseudo frequency is 3.87 years.

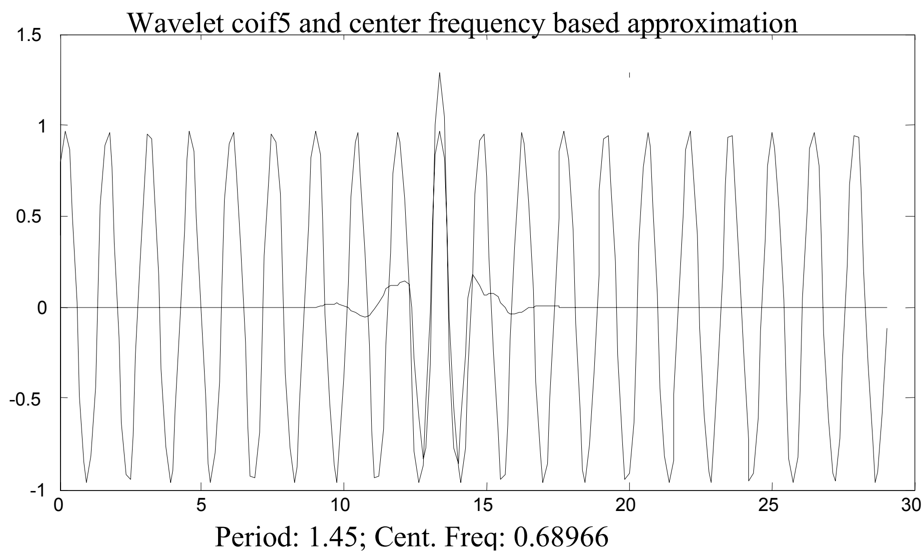

Figure B2: Scale in terms of equivalent sinusoidal frequency

Appendix C: Detailed result tables

The tables that follow give energy decompositions for different versions of the reference currency rates. Note that these are quasi variances for orthogonal details, so to get total energy, just sum across the columns or a selection of columns.

Table C1: Energy table for nominal CRR, centred version

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.9881	0.0814	0.9541	0.1114	0.0758	0.0311	0.0336	0.0247
Canada	0.3188	0.1101	0.3893	0.0891	0.0708	0.0310	0.0168	0.0153
Chile	20.7024	1.9569	0.2270	0.3753	0.1192	0.1004	0.0378	0.0324
Germany	1.3631	0.2958	0.1070	0.2250	0.0925	0.0385	0.0183	0.0143
Japan	6.0133	0.0614	1.1145	0.3927	0.1138	0.1122	0.0300	0.0262
Australia	0.1236	0.0414	0.2212	0.1972	0.0866	0.0590	0.0427	0.0278
South Africa	33.1305	0.5170	0.3484	0.5746	0.1555	0.1276	0.0525	0.0358
Sweden	0.4610	0.0645	0.3215	0.2526	0.1358	0.0487	0.0280	0.0111
Switzerland	2.3703	0.0790	0.1925	0.2842	0.1298	0.0639	0.0256	0.0192
UK	0.9174	0.2612	0.3955	0.0627	0.0769	0.0363	0.0173	0.0171
Singapore	4.6753	0.1226	0.1457	0.1228	0.0113	0.0210	0.0115	0.0072
Norway	0.1925	0.0789	0.0668	0.1299	0.0766	0.0242	0.0154	0.0134
US	0.7852	0.5128	0.5267	0.2372	0.0416	0.0409	0.0176	0.0112

Table C2: Energy table for real CRR, centred version

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.2939	0.0846	1.1388	0.1847	0.0491	0.0277	0.0325	0.0247
Canada	0.5953	0.2137	0.3759	0.0940	0.0785	0.0402	0.0165	0.0158
Chile	0.8249	0.7133	0.1630	0.1598	0.1031	0.0947	0.0403	0.0298
Germany	0.0048	0.0552	0.1787	0.1469	0.0745	0.0367	0.0187	0.0138
Japan	0.3393	0.0399	1.2912	0.3525	0.1060	0.1172	0.0303	0.0256
Australia	0.1381	0.0604	0.4021	0.1841	0.0764	0.0600	0.0440	0.0273
South Africa	1.0634	0.8190	0.3151	0.5943	0.1192	0.1197	0.0517	0.0354
Sweden	3.7693	0.0078	0.1849	0.2600	0.1312	0.0477	0.0275	0.0113
Switzerland	1.2851	0.2561	0.2187	0.2829	0.1338	0.0632	0.0271	0.0195
UK	1.0090	0.2525	0.5442	0.0596	0.0905	0.0394	0.0168	0.0171
Singapore	0.6715	0.1069	0.2124	0.1787	0.0156	0.0195	0.0127	0.0079
Norway	0.1643	0.0924	0.0495	0.1261	0.0750	0.0235	0.0167	0.0128
US	0.1700	0.9079	0.4954	0.2615	0.0401	0.0413	0.0172	0.0109

Table C3: Energy table for nominal CRR with apparent A weights

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.3288	0.2063	1.3065	0.0850	0.0860	0.0439	0.0344	0.0280
Canada	0.7976	0.0622	0.3057	0.1209	0.0779	0.0410	0.0183	0.0190
Chile	32.3483	2.1560	0.2056	0.4602	0.1125	0.1250	0.0407	0.0371
Germany	0.0035	0.3391	0.2435	0.2554	0.0881	0.0330	0.0215	0.0142
Japan	1.7740	0.0303	0.8711	0.2501	0.0917	0.0825	0.0277	0.0221
Australia	0.7792	0.1112	0.3876	0.1862	0.0905	0.0684	0.0399	0.0263
South Africa	47.4601	0.7647	0.5351	0.6577	0.1828	0.1535	0.0619	0.0427
Sweden	3.2970	0.0248	0.4493	0.3717	0.1541	0.0545	0.0338	0.0137
Switzerland	0.1698	0.0591	0.2975	0.3121	0.1309	0.0550	0.0296	0.0186
UK	0.0917	0.1472	0.3703	0.0578	0.0739	0.0338	0.0127	0.0138
Singapore	1.1615	0.2240	0.0605	0.1706	0.0150	0.0230	0.0130	0.0086
Norway	0.5983	0.0514	0.1736	0.1971	0.0906	0.0271	0.0217	0.0155
US	0.0757	0.3952	0.3010	0.2443	0.0464	0.0483	0.0151	0.0134

Table C4: Energy table for nominal CRR with apparent B weights

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.2184	0.2251	1.3777	0.0987	0.0876	0.0340	0.0324	0.0258
Canada	0.4570	0.1140	0.2166	0.0628	0.0654	0.0249	0.0138	0.0141
Chile	29.7400	1.9382	0.2312	0.3399	0.0986	0.1030	0.0369	0.0339
Germany	0.0767	0.4669	0.2714	0.3166	0.0966	0.0480	0.0279	0.0180
Japan	2.4615	0.0707	1.0707	0.3737	0.1199	0.1203	0.0382	0.0312
Australia	0.4183	0.1650	0.3626	0.1770	0.0712	0.0515	0.0314	0.0219
South Africa	44.3325	0.8805	0.5593	0.7467	0.1780	0.1402	0.0656	0.0432
Sweden	2.5043	0.0547	0.3403	0.4000	0.1455	0.0610	0.0389	0.0155
Switzerland	0.4056	0.1183	0.3713	0.3585	0.1432	0.0770	0.0380	0.0233
UK	0.0354	0.0914	0.2466	0.0601	0.0868	0.0407	0.0146	0.0159
Singapore	1.7091	0.2979	0.1157	0.0865	0.0126	0.0160	0.0127	0.0068
Norway	0.2894	0.1068	0.1683	0.1626	0.0952	0.0311	0.0264	0.0178
US	0.0153	0.2810	0.2893	0.1460	0.0330	0.0280	0.0110	0.0085

Table C5: Energy table for real CRR with apparent A weights

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.2332	0.2148	1.5347	0.1873	0.0554	0.0397	0.0324	0.0277
Canada	1.0072	0.1476	0.3035	0.1033	0.0863	0.0540	0.0175	0.0198
Chile	0.5298	0.9002	0.1169	0.1749	0.0938	0.1202	0.0425	0.0340
Germany	0.1187	0.2130	0.3074	0.2056	0.0754	0.0330	0.0214	0.0139
Japan	0.3823	0.0416	0.9781	0.2387	0.0932	0.0857	0.0287	0.0216
Australia	0.2122	0.1704	0.6030	0.1609	0.0733	0.0651	0.0408	0.0254
South Africa	1.7149	1.3132	0.5100	0.6819	0.1374	0.1474	0.0614	0.0420
Sweden	4.9216	0.0482	0.3231	0.3943	0.1470	0.0541	0.0331	0.0141
Switzerland	0.7473	0.4779	0.3374	0.3612	0.1403	0.0570	0.0315	0.0190
UK	0.5384	0.0733	0.5496	0.0702	0.0818	0.0371	0.0127	0.0138
Singapore	0.5290	0.2079	0.1172	0.1890	0.0183	0.0219	0.0143	0.0093
Norway	0.3848	0.0717	0.1524	0.1777	0.0918	0.0278	0.0236	0.0151
US	0.0207	0.5155	0.2690	0.2211	0.0458	0.0499	0.0147	0.0133

Table C6: Energy table for real CRR with apparent B weights

	A7	D7	D6	D5	D4	D3	D2	D1
Average period (Years)	15.4700	15.4667	7.7333	3.8667	1.9333	0.9667	0.4833	0.2417
New Zealand	0.1635	0.2966	1.5352	0.2291	0.0619	0.0298	0.0306	0.0255
Canada	0.9825	0.1646	0.2041	0.0572	0.0742	0.0359	0.0131	0.0149
Chile	0.6180	0.9941	0.1171	0.1170	0.0816	0.1003	0.0398	0.0307
Germany	0.1411	0.3182	0.3677	0.2467	0.0799	0.0471	0.0272	0.0177
Japan	0.4980	0.0764	1.2078	0.3462	0.1214	0.1247	0.0395	0.0306
Australia	0.1740	0.2544	0.5278	0.1609	0.0565	0.0493	0.0324	0.0209
South Africa	1.7825	1.5596	0.5097	0.8057	0.1422	0.1339	0.0652	0.0428
Sweden	5.0127	0.1010	0.2306	0.4047	0.1393	0.0608	0.0380	0.0158
Switzerland	0.7514	0.6173	0.3992	0.3995	0.1510	0.0783	0.0401	0.0239
UK	0.4744	0.0352	0.3953	0.0732	0.0969	0.0438	0.0146	0.0159
Singapore	0.6494	0.2809	0.2040	0.1033	0.0165	0.0145	0.0137	0.0074
Norway	0.3699	0.1046	0.1232	0.1681	0.0971	0.0313	0.0285	0.0175
US	0.0232	0.3831	0.2815	0.1369	0.0318	0.0289	0.0107	0.0083

Table C7: Total detail energies for CRR's

	Nominal centred	Real centred	Nominal weights A	Nominal weights B	Real weights A	Real weights B
New Zealand	1.3121	1.5421	1.7902	1.8813	2.0920	2.2087
Canada	0.7224	0.8346	0.6450	0.5115	0.7321	0.5640
Chile	2.8490	1.3039	3.1372	2.7819	1.4826	1.4806
Germany	0.7914	0.5245	0.9948	1.2453	0.8697	1.1045
Japan	1.8508	1.9627	1.3755	1.8247	1.4876	1.9466
Australia	0.6759	0.8544	0.9101	0.8805	1.1389	1.1021
South Africa	1.8114	2.0543	2.3984	2.6136	2.8932	3.2590
Sweden	0.8622	0.6705	1.1019	1.0557	1.0140	0.9902
Switzerland	0.7942	1.0013	0.9028	1.1297	1.4243	1.7092
UK	0.8670	1.0203	0.7096	0.5561	0.8384	0.6749
Singapore	0.4421	0.5537	0.5148	0.5483	0.5779	0.6403
Norway	0.4052	0.3959	0.5771	0.6082	0.5601	0.5704
US	1.3880	1.7742	1.0639	0.7968	1.1294	0.8812

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Endnotes

[1] In terms of the development that follows, suppose a_1, a_2, a_3 are a set of reference rates in a three-country world. The no arbitrage matrix of bilateral rates will be of the form $S = \begin{bmatrix} 0 & a_1 - a_2 & a_1 - a_3 \\ & 0 & a_2 - a_3 \\ & & 0 \end{bmatrix}$, with the elements below the diagonal filled in from $S' = -S$. But this matrix contains the same information as $\begin{bmatrix} 0 & b & c \\ & 0 & c - b \\ & & 0 \end{bmatrix}$ with only two independent elements b,c. Imposing a normalisation such as $a_3 = 0$ or $a_1 + a_2 + a_3 = 0$ resolves the indeterminacy.

[2] For instance, one could choose the elements of ***w*** as proportional to the current account balances, collectively ***x***, of the respective countries, measured in their own currencies. In an entire world it should be true that $\mathbf{a}'\mathbf{x} = 0$; countries that run a positive current account finance those with a negative one. But this is also the condition for a reference basket price against itself. For such a choice of **w,** the USD would be short in the reference portfolio or basket, and the JY long.

[3] The Chilean central bank has been using a reference rate against a basket of currencies but this is adjusted from time to time and the band limits at any time are also fairly generous.

[4] A large volume of NZ borrowing is done via NZD denominated Eurobond and Uridashis. But the ultimate investor source currencies are mainly the Euro, the USD, and the JY. See Bowden (2005).