Neutron reflection interferometry: 
Extraction of the phase in total reflection from stratified media

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Received 9 March 1995

Abstract

Lloyd's mirror configuration has been suggested for determination of the reflection phase, in order to obtain information about the reflecting stratification. For thermal neutrons total reflection occurs at small glancing angles. We show that the phase of the reflection amplitude in total reflection is linear in the glancing angle at small angles, and determine the coefficient of proportionality for three solvable variations of the scattering length density. The semiclassical reflection phase is shown to fail at grazing incidence, but, for smooth profiles which are thick compared to the neutron wavelength, may have a useful range of glancing angles smaller than the critical angle.

1. Introduction

Lloyd's mirror experiment [1] produces interference fringes between a direct and a reflected beam, providing information about their relative phase. Klein, Opat and collaborators have suggested Lloyd's mirror configuration for neutrons [2], and implemented it for light [3]. They used the semiclassical short-wave limit for the reflection phase

$$\delta \approx \delta_s = 2 \int_0^{z_0} dz \, q(z) - \pi/2,$$  \hspace{1cm} (1)

where $q(z)$ is the local value of the normal component of the wave vector, and $z_0$ is the classical turning point at which $q$ is zero (see for example Ref. [4, Sections 6–7 and A-6, Eq. (113)]). This approximation is known to fail at grazing incidence, where the effective wavelength $2\pi/q$ is large; see Figs. 6–9 and the accompanying discussion in Ref. [4]. Since the Lloyd mirror experiment for neutrons must be performed at glancing angles $\theta$ smaller than the critical angle $\theta_c$ for total reflection, and $\theta_c$ is typically of the order of one degree for neutrons of 10 Å wave-length [5], we need to examine the behaviour of the reflection phase $\delta$ at small glancing angles. We shall show that, in all cases,

$$\delta = \pm \pi + c\theta + O(\theta^3),$$  \hspace{1cm} (2)

where the coefficient $c$ is determined mainly by the variation in scattering length density at the outer boundary of the reflecting stratification. The initial variation in the reflecting medium is important at glancing incidence, because the neutron wave penetrates only to a distance of order $z_0$, which goes to zero at grazing incidence.

At grazing incidence ($\theta \to 0$) Eq. (2) correctly gives the reflection amplitude $r = e^{i\delta} \to -1$, in agreement with the general result of Ref. [4, Section 2–3], whereas Eq. (1) gives $r \to -i$. As we shall see, the short-wave formula (1) also fails to give the correct $\theta$-dependence at small $\theta$. 

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SSDI 0921-4526(95)00425-4
2. A general formula for the total reflection phase

Let neutrons of wavelength $\lambda$ be incident from vacuum onto a stratified layer situated between $z = 0$ and $z = \Delta z$. The scattering length density $\rho = b/v$ ($b$ is the bound scattering length, and $v$ is the volume per scatterer) is assumed to be a function of $z$ only. Then the wave equation for the neutrons separates: if the neutrons travel in the $zx$ plane, the wave function is (approximately) the modulated plane wave $\exp(iKx)\psi(z)$, where $\psi$ satisfies

$$\frac{d^2\psi}{dz^2} + q^2(z)\psi = 0,$$

$$q^2(z) = q_1^2 - 4\pi\rho(z),$$

with

$$q_1 = \frac{2\pi}{\lambda} \sin \theta, \quad K = \frac{2\pi}{\lambda} \cos \theta,$$

being the normal and tangential components of the wave vector in the medium of incidence. Because of the translational invariance in the $x$-direction, $K$ is a constant of the motion, while $q(z)$ is not. The classical turning point $z_0$ is given by $q(z_0) = 0$, which implies

$$q_1^2 = 4\pi\rho(z_0) \quad \text{or} \quad \rho(z_0) = \frac{\sin^2 \theta}{\lambda^2}.$$

The $z$-dependence of the wave function can be written as

$$\psi(z) = \begin{cases} 
\exp(iq_1z) + r\exp(iq_1z), & z \leq 0, \\
\alpha F(z) + \beta G(z), & 0 < z < \Delta z, \\
\exp(-iq_1z), & z \geq \Delta z,
\end{cases}$$

where $r$ and $t$ are the reflection and transmission amplitudes, and $F(z)$ and $G(z)$ are linearly independent solutions of Eq. (3) within the reflecting layer. The substrate is assumed to be homogeneous, with scattering length density $\rho_2$, so that

$$q_2^2 = q_1^2 - 4\pi\rho_2 \equiv q_0^2 - q_1^2.$$

The critical angle $\theta_c$ is defined by $q_2 = 0$, so that

$$\sin^2 \theta_c = \frac{\lambda^2}{2\rho_2}.$$

For $\theta < \theta_c$ the neutrons are totally reflected, with exponential decay (evanescence) into the substrate according to $\exp(-|q_2|z)$, where

$$|q_2| = (4\pi\rho_2 - q_1^2)^{1/2} = \frac{2\pi}{\lambda} (\sin^2 \theta_c - \sin^2 \theta)^{1/2}.$$

The continuity of $\psi$ and $\frac{d\psi}{dz}$ at $z = 0$ and $z = \Delta z$ gives four equations, which can be solved for the four unknowns $r$, $t$, $\alpha$, and $\beta$ in Eq. (6). We find (cf. Eq. (2.25) of Ref. [4]):

$$r = q_4 q_2(F, G) + iq_4(F, G') + iq_2(F', G) - (F', G'),$$

$$q_2 q_4(F, G) + iq_4(F, G') - iq_2(F', G) + (F', G').$$

where

$$(F, G) \equiv F_1 G_2 - G_1 F_2,$$

$$(F, G') \equiv F_1 G_2' - G_1 F_2',$$

$$(F', G') \equiv F_1' G_2' - G_1' F_2,$$

and $F_1 = F(0+)$, $F_2 = F(\Delta z -)$, $F_1' = dF/dz$ evaluated at $z = 0+$, etc. We see that $r \to -1$ as $q_1 \to 0$, as stated above. When $\theta < \theta_c$, $q_2 = i|q_2|$ and

$$r = \frac{q_4 [(F, G') + |q_2|(F, G)] + i[(F', G') + |q_2|(F', G)]}{q_4 [(F, G') + |q_2|(F, G)] - i[(F', G') + |q_2|(F', G)]},$$

The wave equation (3) is linear, with real coefficients. Thus the functions $F(z)$ and $G(z)$ may be chosen to be real, in which case the phase of $r$ ($r = e^{i\delta}$ for $\theta < \theta_c$) is given by

$$\delta = 2 \arctan \left\{ \frac{(F', G') + |q_2|(F', G)}{q_4 [(F, G') + |q_2|(F, G)]} \right\}.$$

Since

$$\arctan(X) = \frac{\text{sgn}(X)}{2} \pi^{-1} - X^{-1} + O(X^{-3}),$$

$$\sin^2 \theta_c = \frac{\lambda^2}{2\rho_2}.$$
we see that
\[ \delta = \pi \text{sgn}(Q_0) - \frac{2q_1}{Q_0} + O\left(\frac{q_1}{Q_0}\right)^3 \]  \hspace{1cm} (15)\]
with \(Q_0\) equal to the limiting value of \(Q\) as \(q_1 \to 0\) at grazing incidence, where, from Eq. (9),
\[ |q_2| \to q_c = (4\pi \rho_2)^{1/2} = \frac{2\pi}{\lambda} \sin \theta_c \]  \hspace{1cm} (16)\]
(for \(q_1 < q_c\) we have \(|q_2| = (q_c^2 - q_1^2)^{1/2}\)). Since \(q_1 = \frac{2\pi}{\lambda} \sin \theta\), Eq. (15) proves Eq. (2) for reflecting profiles of finite thickness.

We now look at some examples of the behaviour of the phase \(\delta\).

3. Exact total reflection phase for three profiles

We will give explicit formulae for the phase \(\delta\) in total reflection, i.e. for \(q_1 < q_c = (4\pi \rho_2)^{1/2}\), for the three variations of the scattering density \(\rho(z)\) shown in Fig. 1.

3.1. Homogeneous layer

When \(\theta < \theta_a\) where \(\sin^2 \theta_a = \lambda^2 \rho_a / \pi, q = (2\pi/\lambda) \times (\sin^2 \theta_a - \sin^2 \theta)^{1/2} = |q|\) and the solutions of the wave equation within the layer are \(F(z) = e^{-iqz}\) and \(G(z) = e^{+iqz}\). Then, with
\[ C = \cosh |q| \Delta z, \quad S = \sinh |q| \Delta z, \]  \hspace{1cm} (17)\]
we have
\[ (F, G) = 2S, \quad (F', G') = -2|q|^2 S, \]  \hspace{1cm} (18)\]
\[ (F, G') = 2|q|C, \quad (F', G) = -2|q|C, \]

Fig. 1. Three scattering density profiles for which the total reflection phase is calculated: (a) homogeneous layer; (b) linear variation of \(\rho(z)\); (c) \(\rho = \rho_2/(1 + e^{-z/a})\).

Thus the function \(Q\) defined in Eq. (13) is
\[ Q = -|q| \left( \frac{T + |q_2|/|q|}{1 + T|q_2|/|q|} \right), \]  \hspace{1cm} (19)\]
where \(T = \tanh(\lambda \Delta z)\). As \(q_1 \to 0, |q_2| \to q_c\) and \(|q| \to q_a = (2\pi/\lambda) \sin \theta_a\), so
\[ \lim_{q \to q_a} Q = -q_a \frac{\tanh(q_a \Delta z) + q_c/q_a}{1 + (q_c/q_a)\tanh(q_a \Delta z)}. \]  \hspace{1cm} (20)\]
This varies between \(-q_a\) (when \(q_a \Delta z \ll 1\)) and \(-q_a\) (when \(q_a \Delta z \gg 1\)). If \(\rho_a < \rho_2\) and \(\theta_a < \theta < \theta_c, q = (2\pi/\lambda) (\sin^2 \theta - \sin^2 \theta_a)^{1/2}\) is real and the appropriate solutions are \(F(z) = \cos qz\) and \(G(z) = \sin qz\). Then with
\[ c = \cos q \Delta z, \quad s = \sin q \Delta z, \]  \hspace{1cm} (21)\]
we find
\[ (F, G) = s, \quad (F', G') = q^2 s, \]  \hspace{1cm} (22)\]
\[ (F', G) = -qc. \]

The function \(Q\) which gives \(\delta = 2 \arctan(Q/q_1)\) is now
\[ Q = q \tan[q \Delta z - \arctan(|q_2|/|q|)]. \]  \hspace{1cm} (23)\]

At \(\theta = \theta_a\) the normal component of the wave vector within the layer is zero, and both Eqs. (19) and (23) give
\[ Q(\theta = \theta_a) = \frac{-1}{\Delta z + 1/|q_2(\theta_a)|}. \]  \hspace{1cm} (24)\]

From Eq. (20) we conclude that \(Q_0\) is negative, so at grazing angles of incidence the reflection phase increases linearly with \(\theta\) from \(-\pi\). When \(\rho_a = \rho_2\) the layer does not reflect (it is identical to the substrate as far as the neutrons are concerned). Then \(|q| = |q_2|\) and we regain the bare substrate phase
\[ \delta = -2 \arctan(|q_2|/q_1), \]  \hspace{1cm} (25)\]
in accord with the reflection amplitude \((q_1 - i|q_2|)/(q_1 + i|q_2|)\).

From Eqs. (24) and (13) we see that
\[ \delta(\theta_a) = 2 \arctan \left[ \frac{-1}{q_a \Delta z + q_a/(q_c^2 - q_a^2)^{1/2}} \right]. \]  \hspace{1cm} (26)\]
Thus $\delta(\theta)$ changes by at most $\pi$ as $\theta$ increases from zero to $\theta_c$. It follows that if many Lloyd fringes are to be seen, they have to be in the region $\theta_a < \theta < \theta_c$, where the neutron wave function is oscillatory within the layer (we assume that $\rho_a < \rho_2$). The parameter which determines the number of fringes is, from Eq. (23),

$$q(z) = \left[4\pi(\rho_2 - \rho_a)\right]^{1/2} \Delta z,$$

(27)

since $q$ increases from zero to $(q_a^2 - q_c^2)^{1/2}$ as $\theta$ increases from $\theta_a$ to $\theta_c$. An additional fringe appears for each increment of this parameter through an odd multiple of $\pi/2$; thus a count of the fringes gives an estimate of the quantity (27) (see Section 4 for a discussion of other contributions to the total phase). Fig. 2 shows the total reflection phase $\delta$ for a layer of silicon on iron.

### 3.2. Linear profile

We now consider the scattering length density profile shown in Fig. 1(b). When $\rho(z)$ is linear in $z$, so is $q^2(z) = q_1^2 - 4\pi\rho(z)$. We change to

$$\frac{d^2\psi}{dz^2} + \zeta^2 \psi = 0$$

(29)

with solutions $Ai(-\zeta)$ and $Bi(-\zeta)$, the Airy functions [7]. These functions, and their derivatives, need to be evaluated at the end points

$$\zeta_a = (q_1^2 - 4\pi\rho_a)/k^2, \quad \zeta_b = (q_1^2 - 4\pi\rho_b)/k^2.$$  

(30)

For definiteness we will assume $0 < \rho_a < \rho_b < \rho_2$ in the discussion that follows. At grazing incidence both $\zeta_a$ and $\zeta_b$ are negative, and the functions $F(z) = Ai(-\zeta)$ and $G(z) = Bi(-\zeta)$ respectively decrease and increase monotonically from $z = 0$ to $\Delta z$. At $\theta_a$ defined by $\sin 2\theta_a = \sqrt{\rho_2 - \rho_a}/\rho_a$ the end point $\zeta_a$ is zero. For $\theta_a < \theta < \theta_b = \arcsin(\lambda^2\rho_b/\pi)^{1/2}$ the functions $F$ and $G$ change from oscillatory to monotonic behaviour at the classical turning point $z_0$ defined in Eq. (5). The turning point moves from $z = 0$ to $z = \Delta z$ as $\theta$ increases from $\theta_a$ to $\theta_b$. Before the turning point the Airy functions are, for large argument, proportional to $\zeta^{-1/2}$ times the sine or cosine of $\frac{2\xi}{3}n^{3/2} + \frac{\pi}{2}$. From Eq. (20),

$$\frac{1}{6\pi \Delta \rho}(q_b^2 - q_2^2)^{3/2} = \frac{4\sqrt{\pi}}{3}(\rho_b - \rho_a)^{1/2} \Delta z$$

(31)

determines the number of Lloyd fringes seen for $\theta \leq \theta_b$ (an additional fringe appears for each increment of this parameter by $2\pi$). We note that, as in the case of the corresponding fringe counting parameter (27) for a homogeneous layer, the fringe number is independent of the neutron wavelength: for shorter wavelengths the same number of fringes is compressed into a narrower angular range.

The short-wave approximation to the phase, $\delta_a = 2\int_0^{z_a} dq(z) - \pi/2$ can be evaluated analytically.
We find

$$\int_{z_0}^{z_e} dz \, q(z) = \begin{cases} 0, & 0 \leq \theta \leq \theta_a, \\ \frac{1}{6\pi} \frac{\Delta \rho}{\Delta \rho} (q_1^2 - q_2^2)^{3/2}, & \theta_a \leq \theta \leq \theta_b, \\ \frac{1}{6\pi} \frac{\Delta \rho}{\Delta \rho} \{(q_1^2 - q_2^2)^{3/2} - (q_1^2 - q_0^2)^{3/2}\}, & \theta_b \leq \theta \leq \theta_c, \end{cases}$$

As $\Delta \rho = \rho_b - \rho_a$ tends to zero, this reduces to the homogeneous layer phase integral

$$\int_{z_0}^{z_e} dz \, q(z) = \begin{cases} 0, & 0 \leq \theta \leq \theta_a, \\ (q_1^2 - q_2^2)^{1/2} \Delta z, & \theta_a \leq \theta \leq \theta_c \end{cases}$$

The exact phase and the short-wave approximate phase are compared in Fig. 3. In calculating the exact phase from Eq. (13) we need derivatives of the Airy functions with respect to $z$. These are related to the $\zeta$ derivatives via

$$\frac{d}{dz} = - \text{sgn} \left( \frac{\Delta \rho}{\Delta \rho} \right)^{1/3} \left( 4\pi \frac{\Delta \rho}{\Delta \rho} \right)^{1/3} \frac{d}{d\zeta}.$$

3.3. Hyperbolic tangent profile

In contrast to the two profiles just considered, the profile

$$\rho(z) = \frac{\rho}{1 + e^{-z/a}} = \frac{1}{2} \rho_2 \left[ 1 + \tanh (z/2a) \right]$$

has no finite boundaries. Since $e^x \approx 20$, $\rho(z)$ varies from about 5% to about 95% of $\rho_2$ as $z$ increases from $-3a$ to $+3a$. From Ref. [8] or directly from the formula (10.44) of Ref. [4] we find for this profile the total reflection phase

$$\delta = 2 \sum_{n=1}^{\infty} \frac{2y_1 y_2}{n(n^2 + 4y_1^2 - y_2^2)}$$

where

$$y_1 = q_1 a,$$

$$|y_2| = |q_2| a = (q_2^2 - q_1^2)^{1/2} a \equiv (y_2^2 - y_1^2)^{1/2}.$$

The leading terms at grazing incidence are (from Eqs. (14) and (36), setting $|y_2| \to q_1 a \equiv y_c$):

$$\delta = - \pi \text{sgn}(\tan \pi y_c)$$

$$+ 2y_1 \left\{ - \frac{\pi}{\tan \pi y_c} + \sum_{n=1}^{\infty} \frac{2y_2}{n(n^2 - y_c^2)} \right\} + O(y_c^3).$$

The infinite sum in Eq. (38) may be expressed in terms of the psi function $\Psi = \Gamma'/\Gamma$, since (Ref. [7, Eq. (6.3.16)])

$$\Psi(1 + z) = - \gamma + \sum_{n=1}^{\infty} \frac{n}{n(n + z)},$$

where $\gamma = 0.5772\ldots$ is Euler's constant. Thus

$$\sum_{n=1}^{\infty} \frac{2y_2^2}{n(n^2 - y_c^2)} = - \{ 2\gamma + \Psi(1 + y_c) + \Psi(1 - y_c) \}.$$

The apparent singularity at integer $y_c$ is removable: on using the reflection formula (Ref. [7, Eq. (6.3.7)])

$$\Psi(1 - z) = \Psi(z) + \pi \cot \pi z,$$

Fig. 3. Comparison of the exact and semiclassical reflection phases for the linear ramp profile of Fig. 1(b), with $\rho_a = 0.215$, $\rho_b = 0.644$ and $\rho_2 = 0.805$ (units of $10^{-5}$ $A^{-2}$) for a layer thickness $\Delta z = 3182$ (this corresponds to 2000 $\AA$ for $\lambda = 2\pi A$). The full curve is the exact phase, the dashed curve the short-wave approximation.
Eq. (38) reduces to
\[ \delta = -\pi \text{sgn} (\tan \pi \gamma_c) \]
\[ - 2y_1 [2\gamma + \Psi(y_c) + \Psi(1 + y_c)] + O(y_c^3). \]  
(42)

This formula demonstrates that the grazing incidence form (2) of the total reflection phase is not restricted to profiles of finite extent.

The semiclassical phase formula (1) applies to profiles which have \( q(z) = q_\infty \) for \( z \leq 0 \) (see Section A-6 of Ref. [4]). This is not the case for the profile (35); the appropriate short-wave phase has been evaluated in Ref. [4, Eq. (6.129)]. It is
\[ \delta_\alpha = 4y_1 \log \left( \frac{2y_1}{y_c} \right) - \pi/2 - 4 |y_2| \arctan \left( \frac{y_1}{|y_2|} \right). \]
(43)

At the critical angle, the Euler–Maclaurin summation formula (Ref. [7], Eq. (3.6.28)) gives \( \delta(\theta_c) = 4y_c \log 2 - \pi/2 + (4y_c)^{-1} + (96y_c^3)^{-1} + O(y_c^{-5}), \) of which the first two terms are reproduced by \( \delta_\alpha. \) Fig. 4 compares the exact phase (36) with its linear approximation (42) and with the semiclassical phase (43). All the phases depend on the parameter \( y_c = q_\infty a = (4\pi q_2)^{1/2}a. \) For \( \rho_2 = 0.805 \times 10^{-5} \text{Å}^{-2} \) and \( a = 333 \text{Å} \) (the 5% to 95% of \( \rho_2 \) thickness is then about 2000 Å), \( y_c \approx 3.35. \)

4. The total phase difference between direct and reflected rays

In the preceding sections we explored the properties of the reflection phase \( \delta, \) which relates to the motion in the \( z \)-direction, normal to the reflecting stratification. There is also a phase difference between the reflected and direct neutron waves due to motion in the \( x \)-direction. Fig. 5 shows the two paths.

The phase difference between the reflected and direct neutron rays can be broken up into the straight-path contributions (where the neutrons travel in vacuum), and the curved part contribution from within the reflecting stratification. The straight-path phase difference is
\[ \Delta_s = \frac{2\pi}{\lambda} \left\{ \sqrt{L_1^2 + H^2} + \sqrt{L_2^2 + h^2} - \sqrt{L_2^2 + (H - h)^2} \right\}, \]
(44)

The curved part contributes
\[ \Delta_c = K \Delta x + \delta = \frac{2\pi}{\lambda} \Delta x \cos \theta + \delta. \]
(45)

The tangential component \( K \) of the wave vector is a constant of the motion in a planar stratification. The value of \( \Delta x \) in the ray picture is (Ref. [4,
Eq. (10.50])
\[ \Delta x \approx 2 \int_{z_0}^{z_0} dz \cot \theta(z) = 2 \int_{z_0}^{z_0} dz \frac{K}{q(z)} \]
\[ = K \sqrt{\frac{\pi}{\rho(z_0) - \rho(z)}}. \quad (46) \]

The last equality follows from Eqs. (3) and (5),

which imply

\[ q(z) = 4 \pi [\rho(z_0) - \rho(z)]. \quad (47) \]

The semiclassical value (46) may be obtained from
the more general wave formula (Ref. [4, Eq. (10.51)])

\[ \Delta x = -\frac{d\delta}{dK}, \quad (48) \]

which applies when \( \delta \) varies smoothly with \( K \) (thus
the immediate neighbourhood of the critical angle or wavelength is
excluded, because of the \((\theta - \theta_c)^{1/2}\)

or \((\lambda - \lambda_c)^{1/2}\) singularities, which are universal
for profiles of finite range, as shown in Section 10-2 of
Ref. [4]). On using Eq. (1) in Eq. (48), with \( q(z_0) = 0 \)
and

\[ q(z) = \left(\frac{2\pi}{\lambda}\right)^2 - K^2 - 4\pi \rho(z), \quad (49) \]

we regain Eq. (46). We have seen that Eq. (1) fails at
grazing incidence, and is generally poor when there
are discontinuities in the scattering density. Similar
remarks apply to the corresponding formula (46)
for \( \Delta x \).

The quantity \( 2H h/L \) is the leading term in the
expansion of the geometric path difference in
powers of \( L^{-1} \). The leading terms in \( \Delta \) which depend on the reflecting profile are

\[ \Delta_{\text{profile}} = \delta + (K - 2\pi/\lambda) \Delta x. \quad (50) \]

Since \( K = (2\pi/\lambda) \cos \theta \), the small-angle expansion
of this phase is

\[ \Delta_{\text{profile}} = \pm \pi + c\theta - (\pi/\lambda) \theta^2 \Delta x + O(\theta^3). \quad (51) \]

From Eq. (48) and \( K = (2\pi/\lambda)(1 - \theta^2/2) + O(\theta^4) \)
we have, using Eq. (2),

\[ \Delta x = -\frac{d\delta}{dK} = -\frac{d\delta}{d\theta} \frac{d\theta}{dK} = \frac{c\lambda}{2\pi \theta} + O(\theta). \quad (52) \]

Thus the effect of the transverse displacement on
the phase is opposite to that of the reflection phase,
in the term proportional to \( \theta \):

\[ \Delta_{\text{profile}} = \pm \pi + \frac{1}{2} c\theta + O(\theta^3). \quad (53) \]

5. Summary and discussion

The semiclassical phase formula (1) fails at grazing
incidence, both in value at \( \theta = 0 \) and in the
\( \theta \)-dependence for small \( \theta \).

For profiles of strictly finite range, the formula
(15) gives the small-angle variation, and evaluates
the constant \( c \) in \( \delta = \pm \pi + c\theta + O(\theta^3). \)

The semiclassical phase becomes accurate away
from grazing incidence, provided the profile thickness is large compared to the neutron wavelength,
and the profile is smooth.

The total phase difference has a term proportional
to the lateral displacement of the neutrons
during reflection, \( \Delta x \). This term may be approximated
by a semiclassical path integral, but only to the
same accuracy as that of the semiclassical phase.

At grazing incidence the total profile phase has
the same form as \( \delta(\theta) \), with \( c \) reduced to \( c/2 \).

The Lloyd fringes in a given experiment are likely
to be very closely spaced unless the detector is
very far from the reflecting sample. For example, the contribution \( \delta \) to the phase, in the case of
homogeneous layer of 2000 Å thickness shown in
Fig. 2, increases by five multiples of \( 2\pi \) between 0.3°
and 0.5°. If a photographic film were to adjoin
a layer impregnated with boron 10 or lithium 6, the
neutron absorption reactions \( ^{10}\text{B}(n, \gamma) \) or \( ^6\text{Li}(n, \gamma) \)
could be used to produce a latent image in the film,
with a spatial resolution of the order of 10 μm or better. This may be a more convenient method
of recording Lloyd mirror fringes than the use of
conventional neutron detectors.

References

Section 13.8.