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Long-run Fiscal Projections under Uncertainty: The Case of New Zealand*

Christopher Ball†, John Creedy‡and Grant Scobie§

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Abstract

This paper introduces uncertainty into a fiscal projection model which incorporates population ageing along with a number of feedback effects. When fiscal policy responds in order to achieve a target debt ratio, feedback effects modify the intended outcomes. The feedbacks include the effect on labour supply in response to changes in tax rates, changes in the country risk premium in response to higher public debt ratios, endogenous changes in the rate of productivity growth and savings. Stochastic projections of a range of policy responses are produced, allowing for uncertainty regarding the world interest rate, productivity growth and the growth rates of two components of per capita government expenditure. The probability of exceeding a given debt ratio in each projection year, using a particular tax or expenditure policy, can then be evaluated. Policy implications are briefly discussed.

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1 Introduction

It has long been recognised that the provision of useful policy advice benefits from the construction of projections which describe the possible paths of relevant variables, under clearly stated assumptions. In New Zealand, this is a formal requirement of the Public Finance Act 1989, which requires Treasury to produce a statement on the Crown's long-term fiscal position at least every four years. These statements provide 40-year projections, identify challenges that are likely to face future governments, such as those arising from society's ageing population.[1] Such projections are crucial in assessing fiscal sustainability and the required adjustments in the face of projected debt growth: the issues are discussed by Buckle and Cruickshank (2014) in the New Zealand context.[2]

Projections are obviously subject to a number of limitations. For example, in reviewing the Treasury's modelling work and its Long Term Fiscal Statement (LTFS), Ter-Minassain (2014, p. 50) suggested that, 'the Treasury should continue to refine its analytical tools' and 'it would be desirable to present, in future versions of the LTFS, scenarios with different dynamic paths of the key macroeconomic assumptions, to allow for plausible feedbacks from the growth of the debt'. In this spirit, a long-term model incorporating a small number of feedbacks has recently been developed by Creedy and Scobie (2015).

A further serious limitation is the need to deal explicitly with the inevitable and considerable uncertainty involved in making projections, particularly over such a long period. This was stressed in the review of the Treasury's most recent Long Term Fiscal Statement, by the Controller and Auditor General; see Provost (2013). The review argued that:

> 'Although a single projection makes it easier for a reader to understand, I am concerned that the level of uncertainty implicit in the projection may not be readily understood by readers of the 2013 Statement. I would have liked to see the Treasury make more use of sensitivity analysis in the 2013 Statement to demonstrate this inherent uncertainty.'

However, sensitivity analysis, especially where a range of variables is involved, does

[1] For previous reports see The Treasury (2006, 2009, 2013a).

[2] Early definitions and measures were proposed by Blanchard *et al.* (1990). For a non-technical discussion of issues, see Schick (2005). The approach adopted by the European Union is set out in detail in European Commission (2006). For an example of its use, see also Kleen and Pettersson (2012). A recent review of approaches is by Pradelli (2012).

not attach a probability to alternative outcomes. For this reason, stochastic projections – making explicit assumptions about the relevant distributions – are preferred.[3]

The aim of the present paper is thus to examine stochastic tax revenue, expenditure and government debt projections building on the model of Creedy and Scobie (2015). Rather than attempting to capture all the details involved in the many types of expenditure and tax, that model uses a much more aggregative approach than that of the Treasury's Long Term Fiscal Model.[4] It has only four types of expenditure in addition to debt servicing costs, and a simple proportional income tax function (including interest income). A Goods and Services Tax (GST) is applied uniformly to all consumption expenditure. Various feedback effects are modelled using reduced-form specifications rather than explicit optimising behaviour. The model nevertheless contains a sufficient amount of detail to enable a range of policy responses to be examined.

The basic model is set out briefly in Section 2; further details are in Appendix A. As it would be far too cumbersome to allow all variables in the model to be stochastic, just four key variables of the model are subject to uncertainty. These are two (of the four) expenditure components, the world interest rate and the rate of productivity growth. The approach involves using information about variability in the past to form the basis of expectations about future variability.

Section 3 presents information about the values of these four components of the model over time. This section also describes the method used to carry out the Monte Carlo studies. The 'benchmark' projections, where no policy changes are imposed over the projection period, are presented in Section 4. The model is used in Section 5 to examine the implications of adopting several income tax policy changes designed to achieve a specified debt ratio by the end of the forty year projection period. Comparisons are made with results using the deterministic version of the model. Section 5 examines the implications of a higher productivity growth rate. Conclusions are in Section 6.

The advantage of the stochastic projections is that it is possible to form probability statements about ranges of the debt ratio in each year, in particular the probability that any given debt ratio is exceeded. Such probability statements are, of course, conditional on the model and the specification of uncertainty being correct. It should be recognised

[3]One component of the fiscal projections, stochastic population and social expenditure projections for New Zealand were first made by Creedy and Scobie (2005), and the methodology was followed more recently by Creedy and Makale (2014).

[4]For a description of the Treasury model and projections, see Bell and Rodway (2014).

that a much wider range of uncertainties are relevant in practice. Furthermore, population projections are themselves subject to uncertainty, although deterministic projections are used here. It is sometimes suggested that the discussion of uncertainty surrounding fiscal projections merely provides a distraction from the 'main message' – usually seen as the need for immediate action. However, it is argued here that recognition of the considerable uncertainty involved is indeed a crucial part of the 'message' and itself raises important policy questions. These are discussed briefly in the concluding section.

2 A Description of The Model

A requirement of the model is that it is capable of projecting the paths of government revenue and expenditure, and therefore debt, under a range of assumptions and feedback effects. To make the model as transparent as possible, a high level of aggregation is used. It is clearly necessary to allow demographic variations in both population size and its age composition to influence government expenditure and revenue. While detailed demographic projections are used, distinctions are drawn only between those of 'working age', 'retirement age' and those below working age.

The basic structure of the model is described in Figure 1: further details are in Appendix A and Creedy and Scobie (2015). The shaded boxes indicate components that influence other variables: these include the income tax and GST structure, the incentive effect of taxes, the demographic structure of the population, and productivity and expenditure growth rates. The items in bold font are, along with the debt levels, the main aggregates of total government revenue and expenditure. Feedback effects are indicated by the dashed lines connecting boxes.

The generation of income changes from one year to the next is described down the left-hand side of the diagram which, in turn, leads to tax revenue. There is therefore no attempt to treat the production side of the economy explicitly. The model contains no explicit wage rate, nor does it deal with labour and capital inputs into production. A 'base level' of productivity is taken as exogenously given and, as explained below, productivity changes can arise from growth in public expenditure on health and education per person, which is considered to augment human capital. Government expenditure is described down the central part of the diagram. Starting from per capita values for four expenditure components (welfare benefits for the retired; other welfare benefits per person; expenditure on health

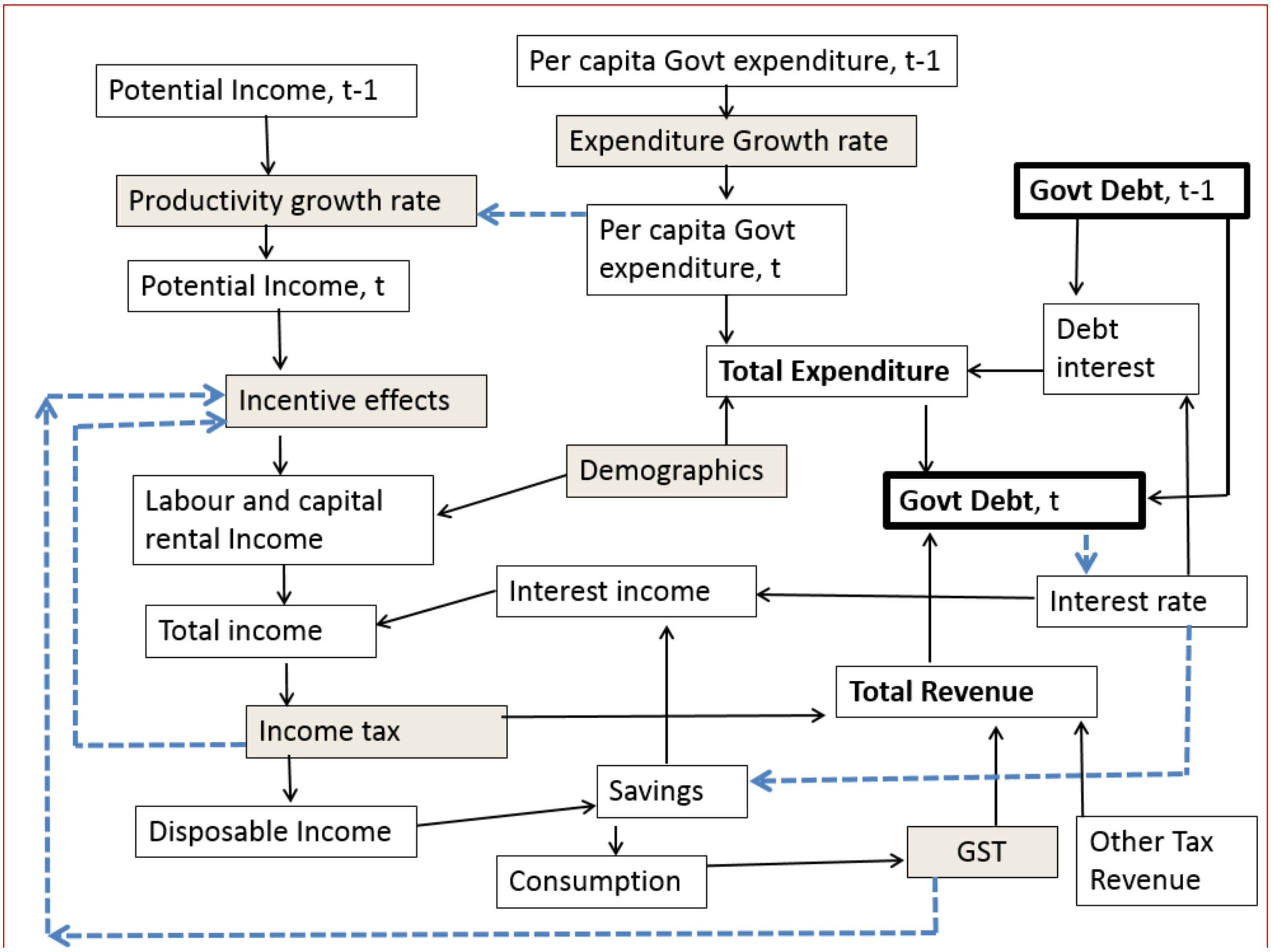

Figure 1: Outline of the Model

and education; other social spending), along with the relevant growth rates, the total value of expenditure is influenced by the age structure of the population. The generation of debt over time is then indicated on the right-hand side of the diagram. All variables are in real terms. The income tax is a simple proportional tax, with constant average and marginal rates, and the GST system is considered to apply to all expenditure.[5]

The four feedback effects modelled are indicated by dashed lines. Taxes have adverse incentive effects which influence employment income. Expenditure on health and education has a (lagged) effect on productivity growth. The government debt ratio influences the interest risk premium. The assumption is made that the same interest rate applies to all debt, so that debt effectively involves the issue of one-period bonds. Finally, the interest rate (the sum of the world rate and the risk premium) affects the saving rate.[6]

The calibration of the model involves setting a large number of initial variables and parameter values, obtained using an extensive range of New Zealand data (see Creedy and Scobie, Tables 2 to 9). An important feature of the model – when the feedback and uncertainty features are 'turned off' – is that it produces 'benchmark' forty-year projections of the government debt ratio that closely match those produced by the considerably more disaggregated Treasury Long Term Fiscal Model. In the benchmark case, growth rates and other policy variables (such as tax rates) are held constant. An absence of feedback effects implies that the economy can be allowed to reach any debt ratio and then brought back to a target level by an appropriate tax and expenditure policy. However, the feedback effects considered here – particularly those affecting the risk premium and labour supply incentives – make recovery from very high debt ratios extremely difficult.

3 Introducing Uncertainty

This section introduces uncertainty into the deterministic form of the model. The structure of the model itself – such as the nature of the basic relationships involved – is not considered to be subject to uncertainty. The parameters of the various reduced-form relationships governing the feedbacks and endogenous changes are assumed to be known and fixed. The

[5]The use of fiscal drag to increase tax revenues over time is thus not a possible policy option in the model. The possibility of the government monetising the debt and inflating is thus also not considered here.

[6]Debt ratios up to about 150 per cent of GDP produce small (linear) increases in the risk premium. Beyond this, the rate increases rapidly (and quadratically). In the stochastic projections reported below, the risk premium was in fact capped at 15 per cent to prevent results becoming too explosive (and thereby distorting the mean and median in particular).

limited nature of the uncertainty modelled therefore needs to be kept in mind, though there are substantial lessons obtained by departing from the deterministic approach that is characteristic of the majority of projection models.

Although the model has a fairly high level of aggregation compared with the Treasury's LTFM, there are nevertheless many variables for which the time path is uncertain. As mentioned in the introduction, the present paper is limited to considering just four variables. These are two of the four expenditure components, the world interest rate and the rate of productivity growth. These were selected for examination on the grounds that they are important determinants of the time path of the debt ratio and have been subject to considerable variability over time. The effect of allowing these four variables to be stochastic is nevertheless sufficient to demonstrate the importance of dealing explicitly with uncertainty.

One approach to dealing with uncertainty is to rely on a set of *a priori* assumptions about the distributions of the relevant variables.[7] Assumptions about the form of the joint distribution of the variables may be based on considerations relating to the structure of the economy, policy settings and vulnerability to a range of exogenous shocks. Such assumption may be more or less informed by past events. For example, the view may be taken that the past is not necessarily a reliable guide to future variability given a range of institutional and other changes. However, this is not the approach adopted here.

In the present context, the question raised is: what are the implications for the likely path of the debt ratio if future variability is thought to be similar to that observed in the past? In the absence of strong reasons for imposing other *a priori* specifications, past history – while obviously not precisely repeated – is regarded as a reasonable starting point. The method used here does not explicitly model discrete catastrophic, or beneficial, events. However, to the extent that such events took place within the past years observed, their effects on the relevant variables are implied. The approach thus assumes that such events are no more or less likely than in the past.

[7] An alternative to stochastic projections can sometimes be used where there is a long series of annual projections, based on data revisions for some variables relating to before the projection period, as well as outcomes of other variables. The variation revealed by such a series provides information about uncertainty associated with projections.

3.1 Past Variations

Historical evidence relating to productivity growth, the world interest rate, and the two expenditure (per person) categories is shown respectively in Figures 2, 3 and Figure 4.[8] The historical time period considered is forty years, the same length of time for which projections are required.

Using these data it is possible to produce frequency distributions of values over the forty-year period. The resulting histograms (not shown here) do not clearly follow any familiar form. One approach may be to use mixture distributions to capture particular features of the distributions. However, in addition to the complexity involved, particularly in dealing with correlations among variables, such distributions would need to be truncated.[9]

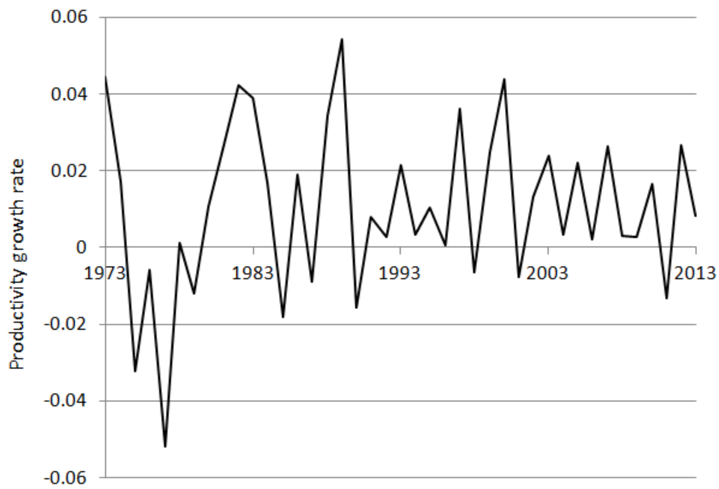

Figure 2: Labour Productivity Growth 1973 to 2013

Table 1 provides summary information about the distributions of the four variables over

[8] Expenditure in nominal terms is obtained from http://www.treasury.govt.nz/government/data. This is converted to real expenditure by deflating by the CPI: see http://www.rbnz.govt.nz/statistics/tables/m1/). Finally, this is converted to a per capita basis using Historical population estimates as at 30 June: 1972-1990: De facto population and 1991-2014: Estimated resident population.

See: http://www.stats.govt.nz/browse_for_stats/population/estimates_and_projections/historical-population-tables.aspx)

Real interest rates are for 10 year US Treasury bonds: see www.economagic.com/fedstl.htm. Labour productivity is defined as GDP per hour worked. Total hours worked seasonally adjusted are from Household Labour Force Survey (lhhwzq) and real GDP (ngdpp_zq) both from Statistics New Zealand, http://www.stats.govt.nz/.

[9] Allowing values to be randomly drawn from the extreme tail of a distribution, however thin, can cause severe problems. In particular, the relationship between the risk premium and the debt ratio produces a rapid increase in the premium for debt ratios exceeding about 1.5 times GDP.

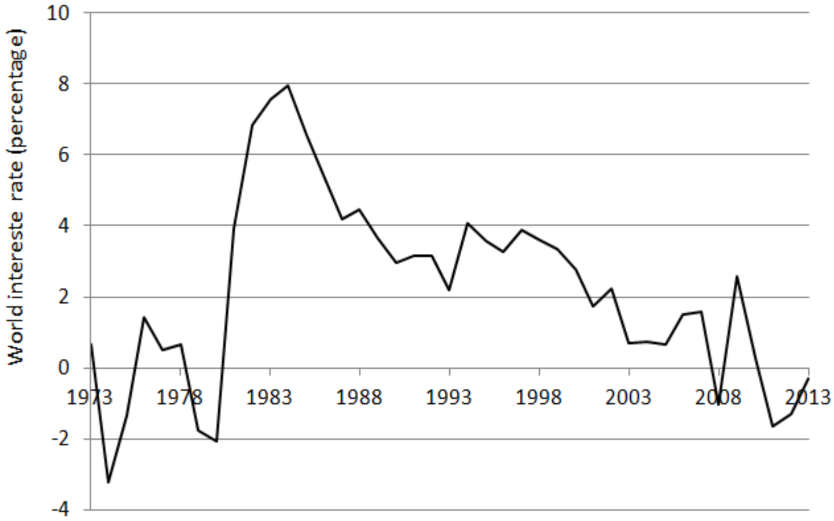

Figure 3: World Real Interest Rate 1973 to 2013

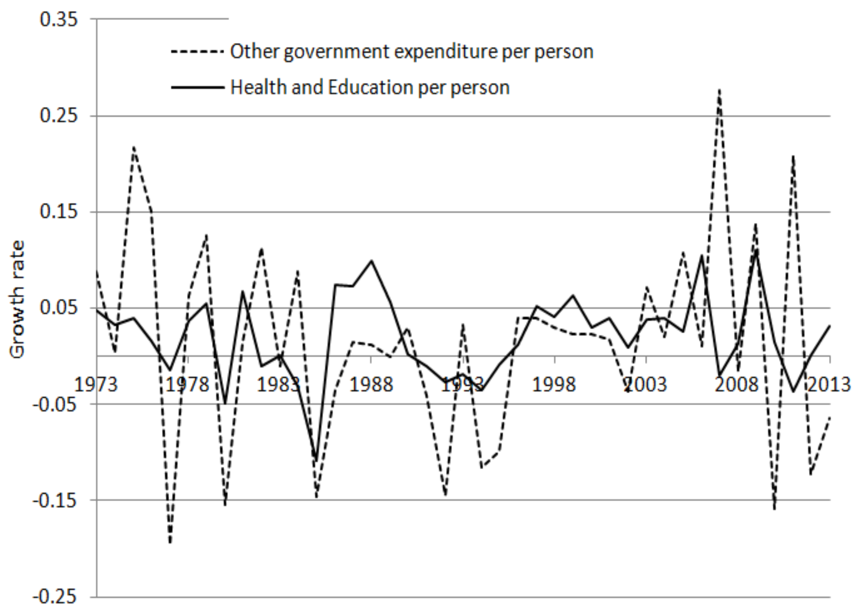

Figure 4: Expenditure Growth Rates 1973 to 2013

the forty year period 1973 to 2013. For a symmetric distribution, skewness (the ratio of the third moment about the mean to the cube of standard deviation) is zero. Hence it can be seen that the growth rates of health and education spending and productivity are negatively skewed while the other two variables are positively skewed. The excess kurtosis measured is relative to the standard normal distribution which has a kurtosis measure of 3; it is thus equal to the ratio of the fourth moment about the mean to the fourth power of the standard deviation, minus 3. Hence health and eduction, other social spending, and productivity growth are more peaked than the normal distribution, while the world real interest rate is flatter than a normal distribution.

Table 1: Summary Measures of Distributions: 1973-2013

	Growth rate of expenditure on		Productivity	World real
	Other social expenditure	Health and Education	growth rate	interest rate (per cent)
Mean	0.0151	0.0209	0.0106	2.1763
Variance	0.0112	0.0020	0.0005	7.0509
Coeff of Variation	7.009	2.042	2.109	1.220
Skewness	0.1514	-0.3000	-0.3850	0.1532
Kurtosis (excess)	0.1208	0.6852	0.6087	-0.2445

Table 2 reports the correlations between the variables. This shows, for example, that two expenditure growth rates are slightly negatively correlated with the world interest rate. More important are the positive correlation between the two growth rates, along with the positive correlation between the growth of expenditure on health and education and the productivity growth rate, and between the latter and the world real interest rate.

Table 2: Correlation Matrix of Distributions: 1973-2013

	Growth rates of:			Word real
	Other social expenditure	Health and education	Productivity	interest rate (per cent)
Other social expend	1	–	–	–
Health and Education	0.2884	1	–	–
Productivity growth	0.0862	0.2104	1	–
World real interest rate (%)	-0.0844	-0.0794	0.2856	1

3.2 Generating Stochastic Variables

In order to capture the complex correlations between variables and the precise forms of their distributions, without having to estimate a fully specified joint distribution, the following approach was used. This was designed to capture the historical variation by sampling from the empirical distributions over the period 1973 to 2013, rather than taking random draws from specific functional forms of the relevant distributions.

First, each empirical distribution was transformed to ensure that the respective geometric means are the same as the parameter values imposed in the deterministic projections examined by Creedy and Scobie (2015).[10] The method then involved taking a random selection of short 'blocks' of years, in moving over the 40 year projection period. The main properties of the empirical data of interest are the cross-correlations among variables and the serial correlation that is also evident in the diagrams. The first property was preserved by selecting data from the same past year for each variable. Hence, if 1985 is selected for one variable, the 1985 values are used for all other variables. The second property was covered by selecting a sequential run of years and randomly drawing a starting point and run length.

Specifically, an initial run length was first obtained by taking a random draw from a uniform distribution over the integers three to seven. Hence each 'block' of years is allowed to vary from between three and seven years inclusive. Then, given the selected run length, the starting year was obtained by taking a random draw from a uniform distribution defined over those starting points which, given the run length, provide years that are completed within the dataset.[11] For example, at the start of the projection period, suppose the first random draw (from between 3 and 7 inclusive) gives 4, while the second draw give the year 1988. The values would be taken from the four (adjusted) distributions in the four years 1988, 1999, 2000, 2001.

In carrying out the Monte Carlo analysis, 2000 sets of stochastic projections were obtained. For each projection year, the arithmetic mean and median of the resulting distribution of debt ratios were calculated, along with the quartiles and the 5^{th} and 95^{th} percentile.

[10] That is, if x_i denotes the value of a particular variable in period i, and α is the value of the variable in the deterministic case (no stochastics), the transformed value is obtained as: $\{(1+\alpha)(1+x_i)/GM(1+x_i)\}-1$. The deterministic case is then equivalent to all $x_i = \alpha$.

[11] Further sensitivity analysis was carried out by sampling using only three and four year blocks of data. The main results are unchanged although there are minor differences in the percentiles.

4 The Benchmark Case

As mentioned above, for the benchmark case where no policy responses take place over the period, the deterministic form of the model produced a projection for the debt ratio in New Zealand that closely matches that of the more disaggregated Treasury Long Term Fiscal Model. This produces the 'expanding debt' case that is recognised as being unrealistic and which corresponds in no sense to a forecast. For this case, the stochastic form of the model gives rise to Figure 5, which shows the time profiles of the mean and median debt ratio, two measures of location, along with various percentiles, The dotted lines are the 5^{th} and 95^{th} percentiles while the dashed lines are the two quartiles. The median value for the final projection year is a debt ratio of 143 per cent of GDP. This is similar to the deterministic case, where Creedy and Scobie (2015) found a debt ratio of (approximately) 150 per cent of GDP in the final year.

The range shown by the 5^{th} and 95^{th} percentiles in the terminal year is vast. The mean is clearly strongly influenced in later years (after about 2033) by the long 'right-hand' tail of the debt ratio distribution as the distributions become more positively skewed as well as more dispersed. This is largely influenced by the dispersion of expenditure as a proportion of GDP, which is influenced by the rapid increase in debt servicing costs in the high-debt ranges, as the risk premium rises steeply. The spread shown in Figure 5 clearly underscores the considerable uncertainty involved in making projections over such a long period.

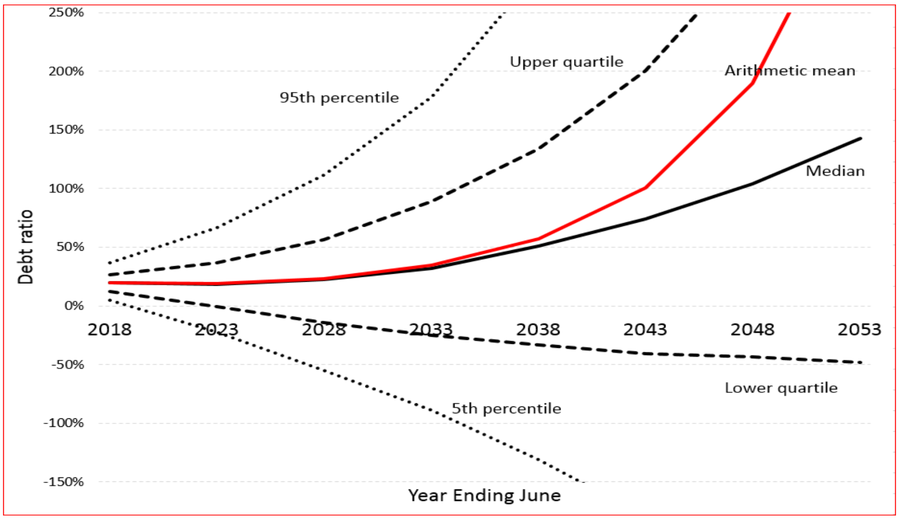

Figure 5: Benchmark Stochastic Projections: No Policy Responses

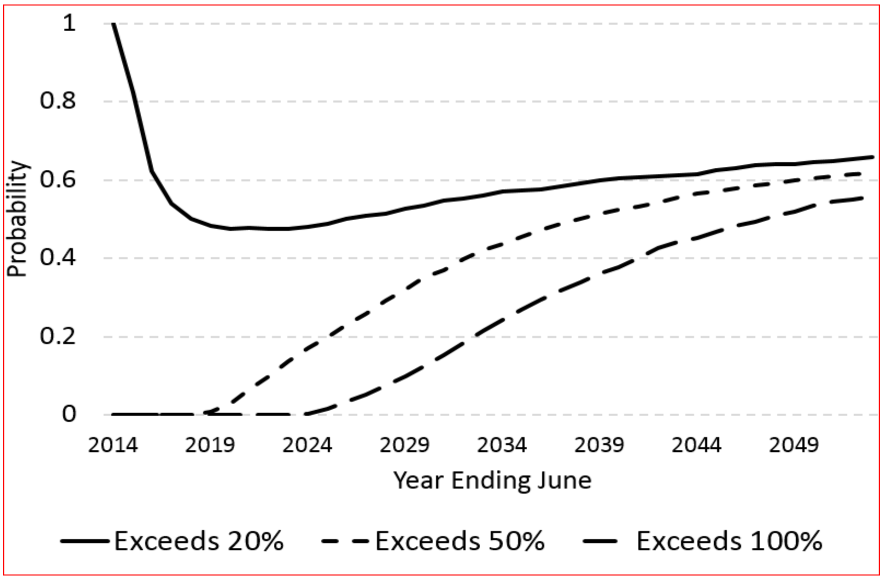

Figure 6: Probability of Exceeding Given Debt Ratios in Each Projection Year

Figure 6 shows the variation over time in the probability of exceeding three debt ratios, of 20, 50 and 100 per cent of GDP. Given that the starting point involves a debt level around 25 per cent of GDP, the probability of exceeding the first ratio is close to 1 in early years, while the probability of exceeding 50 or 100 per cent is zero. But as the dispersion increases, the probabilities of exceeding the higher debt ratios increase, while the probability of exceeding the lower ratio first necessarily falls and then rises. A feature of these probability profiles is that at the end of the projection period the probabilities converge at around a debt ratio of 60 per cent. This arises because of the huge dispersion of the distribution, so that very little of the area under the distribution is contained between the 20 and 100 per cent debt ratio values.

Consider the effect of increasing or decreasing the degree of uncertainty, reflected in the dispersions of the four variables concerned. Figure 7 illustrates two hypothetical distributions of the debt ratio in a particular year, where distribution A reflects more uncertainty than distribution B. These two distributions are shown as symmetric, with increased uncertainty resulting in a mean-preserving spread. In practice the distributions are skewed and the way in which the degree of uncertainty is specified affects the outcome. However, they illustrate a property that holds in the present case. Consider the lower debt ratio, x_L. Letting DR denote the debt ratio and P_A and P_B the probabilities relating to the two distributions, it can be seen that $P_A(DR > x_L) < P_B(DR > x_L)$. However, for the higher debt ratio, x_H, the inequality is reversed and $P_A(DR > x_H) > P_B(DR > x_H)$. Hence the probability of exceeding a relatively low debt ratio falls as uncertainty increases, while the probability of exceeding a relatively high value rises as uncertainty increases.

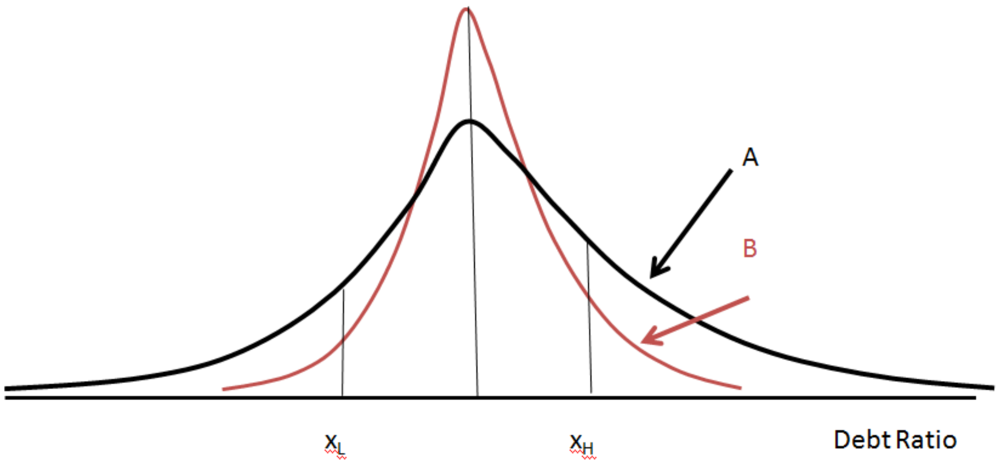

Figure 7: An Increase in Uncertainty

A further implication arises from these comparisons. Suppose the actual degree of uncertainty is greater than is thought; that is, distribution A reflects the actual uncertainty whereas B is generated from the assumptions used to generate the Monte Carlo experiment. In that case, the conclusions will be too pessimistic (attributing too high a probability) about exceeding low debt ratios and too optimistic (attributing too low a probability) about high debt ratios.

As an example, suppose the only uncertainty arises from variations in the world real interest rate which, as seen above, has a relatively low coefficient of variation. Figure 8 shows the corresponding profiles over time of the probability of exceeding the three debt levels considered. All three probabilities approach unity by the end of the period because of the low dispersion of the distribution of the ratio of debt to GDP, with just the one variable being stochastic. The 'dip' in the profile for the probability of exceeding 20 per cent arises because of the slight reduction in the debt ratio profiles in the early years of the projection period.

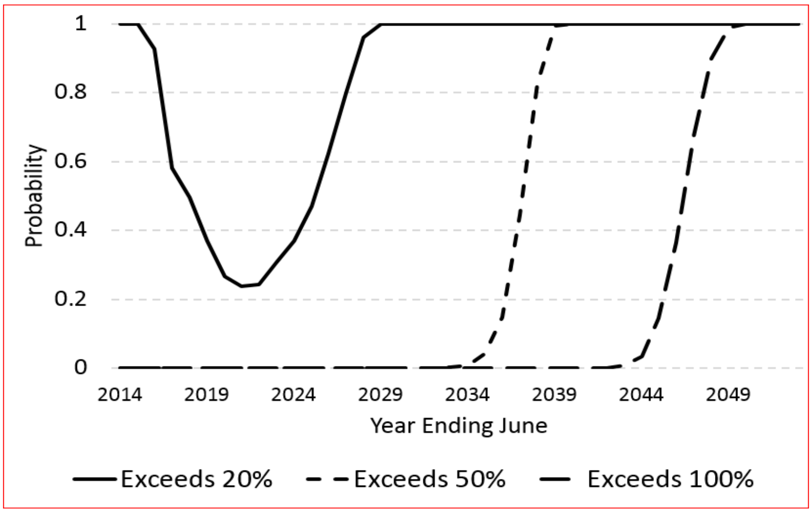

Figure 8: Probability of Exceeding Debt Ratios where Uncertainty Arises from World Interest Rate Only

5 Meeting a Debt Target

It has been stressed that the benchmark projections provide, rather than any kind of forecast, information about fiscal sustainability and an indication of a likely need for future policy changes. An important question – to be examined in future research – concerns the policy implications of providing information about the uncertainty associated with the projections, in addition to the deterministic case. The present section instead examines the implications for the projected time path of the debt ratio of two simple policy responses designed to achieve a specified debt ratio target by the end of the projection period. The first policy, discussed in subsection 5.1, is the widely-discussed tax smoothing option, which implies a period when government surpluses are obtained. The incentive for tax smoothing arises from the fact that the excess burden of taxation increases disporportionately with the tax rate, so that a constant rate minimises the burden. An early modern discussion of tax smoothing is Barro (1979). Armstrong *et al.* (2007) also highlight the concave nature of the government's revenue function, arising from adverse incentive effects. Davis and Fabling (2002) stress the ability of the government to obtain a rate of return, during periods of surplus, in excess of the cost of borrowing, although this feature is not examined here.

The second policy, examined in subsection 5.2, involves using a gradual tax increase to achieve the same target debt ratio. The objective in comparing the policies is not at this stage to consider any type of optimal policy, but rather to compare the time paths of debt and the associated uncertainty, and also to compare results with those obtained when uncertainty is neglected.

5.1 Tax Smoothing

In the case where there is no uncertainty, Creedy and Scobie (2015) found that to achieve a 20 per cent debt ratio in 2053 using income tax smoothing, whereby a higher constant tax rate is imposed over the period, the tax rate needs to be increased to 18.5 per cent from the benchmark income tax rate of 16.25 per cent.[12] If the same policy, of imposing a rate of 18.5 per cent over the period, is imposed, the resulting stochastic projections are shown in Figure 9. As in the benchmark case, the dispersion of the probability distribution rapidly increases over time. The median debt ratio turns out to be about half of the 'desired' 20 per cent. Nevertheless, the median is increasing at the end of the period and would be expected to reach 20 per cent soon afterwards. The associated probabilities of exceeding 20, 50 and 100 per cent of GDP in each year are shown in Figure 10.

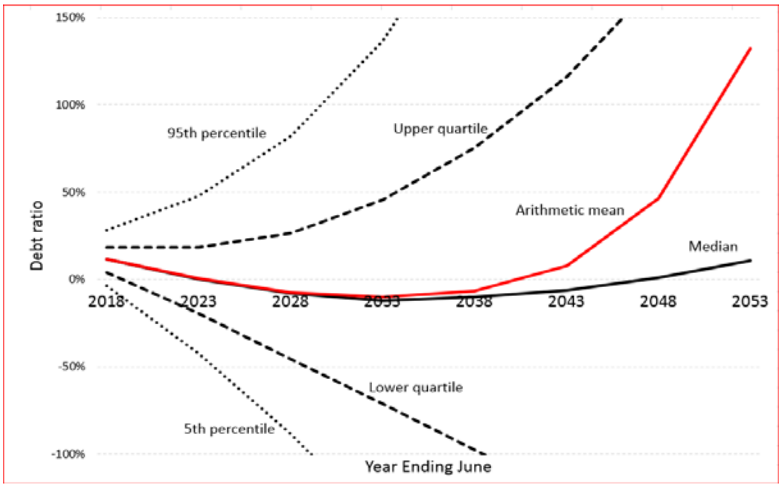

Figure 9: Tax Smoothing with Income Tax Rate of 18.5 Per Cent

An alternative question which the stochastic projections are able to answer concerns how much the constant tax-smoothing income tax rate needs to be increased in order to achieve a specified probability of the debt ratio exceeding various levels. The result of gradually increasing the tax rate is shown in Table 3 for the final projection year, 2053. For example, if the income tax rate were to be increased from the benchmark case of 16.25 per cent to 20

[12] If only GST is adjusted, it needs to be raised, from the benchmark value of 15 per cent, to 18 per cent and held constant over the projection period.

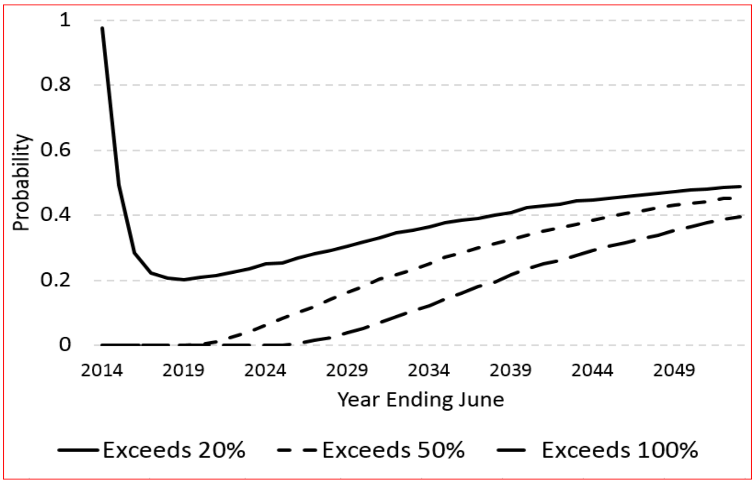

Figure 10: Probability of Exceeding Debt Ratios with Income Tax Rate of 18.5

Table 3: Probability of Exceeding Debt Ratios in 2053 as Income Tax Rate Varies

Tax rate	Ratio of debt to GDP (per cent)			
(per cent)	20	50	100	Mean
16.25	56	52	44	204
17	49	45	38	132
18	41	37	31	46
19	34	30	25	-31
20	27	25	20	-101

per cent, the probability of the debt exceeding 20 per cent of GDP in 2053 would fall from 56 per cent to 27 per cent. This halving of the probability is similar for that of the debt ratio exceeding 50 per cent and 100 per cent of GDP. The relatively small difference in the probabilities, for the different debt ratios, arises for the reason explained above, namely that there is a very large dispersion in the final distribution. However, the associated variation in the expected value of the debt ratio in 2053 is huge: it falls from a ratio of 204 per cent in the benchmark case to a surplus of 101 per cent of GDP with the income tax rate of 20 per cent.

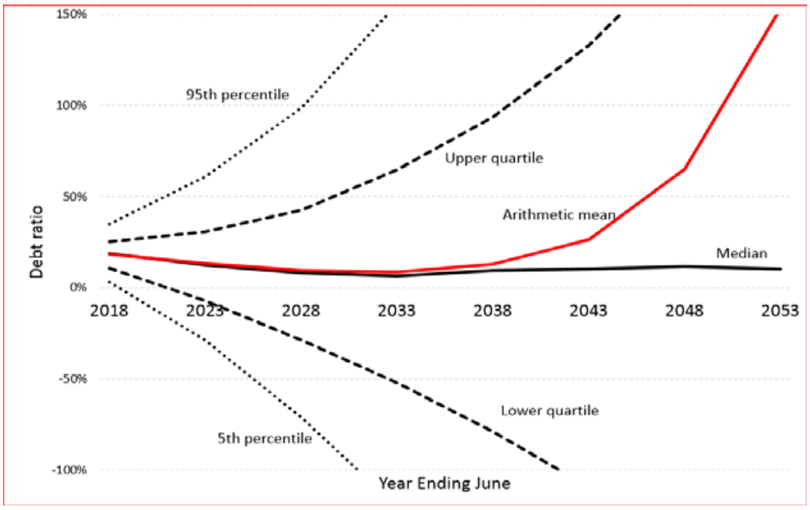

Figure 11: Gradual Tax Rate Increase of 0.14 Percentage Points each Year

5.2 An Increasing Income Tax Rate

In the deterministic case, it was found that a gradual increase in the income tax rate by 0.14 percentage points each year, so that the rate reaches 21.9 per cent in 2053, would result in a debt ratio of 20 per cent of GDP by the end of the projection period. In contrast to the tax-smoothing case which implies a period of surplus, there is less variation in the debt ratio over the projection period. It falls to a low of about 10 per cent of GDP in the middle of the projection period.

If the same policy of gradually increasing the tax rate over time is followed, the resulting stochastic projections are illustrated in Figure 11. In this case the median debt ratio in 2053 is also about half of the value (of 20 per cent) obtained by the deterministic projections. Figure 12 shows the associate probabilities of exceeding three debt ratios in each year. These are similar to those obtained in the tax smoothing case.

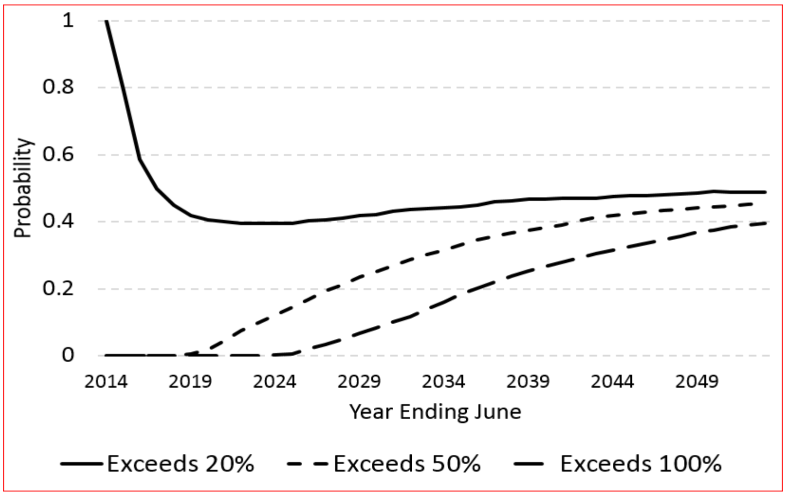

Figure 12: Probability of Exceeding Debt Ratios with Gradually Increasing Income Tax Rate

6 Conclusions

This paper has added uncertainty to the projection model introduced by Creedy and Scobie (2015), which allowed for a limited number of feedback effects in a model designed to examine fiscal sustainability in New Zealand over a long period. These modifications have been made in response to public criticisms of the Treasury's Long Term Fiscal Statement (2014). Faced with the inevitable uncertainty associated with future values of important variables in the model, the approach taken has been to use stochastic projections in a Monte Carlo exercise. This has the substantial advantage over sensitivity analyses of being able to generate probability statements about projected debt ratios.

The approach taken was to suppose that the observed variablity in four central variables (the world interest rate, the base rate of productivity growth, and growth rates of two per capita expenditure categories) over the forty-year period 1973 to 2013, provided a reasonable guide to future variability. Instead of specifying explicit functional forms for the distributions, random observations were taken from 'blocks' (of random length) of the past data. In this way the serial correlations and correlations among the variables were retained. Uncertainty regarding the structure of the projection model itself, and the form of the various feedback relationships (as well as the many other variables which cannot be known with certainty), was ignored. The projected distributions can therefore be regarded as being conditional on these assumptions.

Despite the limited nature of the uncertainty modelled here, it was found that the projections of the debt ratio over the forty-year period to 2053 are subject to extremely large variations, as well as being positively skewed in the later years. The results suggest that the basic deterministic projections provide a very partial view about future prospects, despite not departing substantially from the median values obtained from the stochastic projections.

These results raise the important question of the appropriate policy response. Faced with deterministic projections showing a divergence between expenditure and revenue, and thus an expanding debt ratio, it is sometimes argued that policy responses should be made as early as possible, although such a recommendation is subject to inter-generational equity and other considerations. However, with uncertainty, it is possible that there is an option value of waiting until some of the uncertainty is resolved, particularly where policy changes involve costs that cannot be reversed.[13]

The framework may be compared with the consideration of investment in a multi-period project where the future returns are not known with certainty, there is a non-recoverable sunk cost of investing in the first period and there exists the option of waiting until later periods before making the investment.[14] The sunk cost consists of any fixed costs which cannot be recovered (such as the difference between the cost of investing in specific equipment and its resale value) and the foregone value of waiting and obtaining more information (the option value).[15]

In the present context it has been seen that an increase in the tax rate in the first period, to accumulate a fund that can be used in the event of a future possible expenditure requirement, involves a sunk cost and therefore possibly a positive option value.[16] This sunk

[13] In the present context early tax changes impose costs on a given generation, which cannot necessarily be compensated by future changes. The excess burden of taxation provides a further example. Using a very simple tax and expenditure model, the role of option values was explored by Ball and Creedy (2014).

[14] See, for example, Dixit and Pindyck (1994) and Pindyck (2008).

[15] In the context of health and long-term care under demographic uncertainty, Lassila and Valkonen (2004, p. 637) find that the longer the time horizon, 'the virtues of using continuously updated demographic information to evaluate future expenditures become evident'.

[16] The danger that a precautionary fund will be raided by a future government, stressed long ago by Ricardo in the context of the British Sinking Fund, is not considered here. Davis and Fabling (2002) model 'expenditure creep'and report that it can completely erode the efficiency gains from tax smoothing. They conclude that, 'strong fiscal institutions are a prerequisite for achieving the welfare gains from tax smoothing' (2002, p. 16).

cost arises partly from the nature of the excess burden of taxation which increases more than proportionately with the tax rate. A subsequent reduction in the tax rate, if the extra revenue is not needed, allows the later tax rate to be reduced below the value needed for the other (fully anticipated) tax-financed expenditure. But this cannot fully recover the extra excess burden from the initial tax increase. However, the present context does not require, as in the standard investment framework, a given lumpy amount of investment (such as the construction of a factory) in the period in which it is decided to invest. Here it is possible in the first period to commit to a policy which involves only a small increase in the tax rate, leaving the additional (uncertain) revenue to be obtained by a higher tax rate increase in future, if this turns out to be necessary.

Constraints on flexibility of government policy, such as the ability to change tax rates and expenditure, are also relevant. Auerbach and Hassett (1998, 2002) suggested that, faced with uncertainty, the inability to have frequent policy changes suggests early action. However, inaction may be chosen because of the inability to reverse any adverse effects on particular groups. They concluded (1998, p. 23) that, 'the optimal policy response over time might best be characterized by great caution in general, but punctuated by occasional periods of apparent irresponsibility'. The present model will thus be extended in future research to consider the question of optimal policy responses under uncertainty.

Appendix A: Formal Statement of The Model

This Appendix provides a description of the main components of the model.

1 Government Debt

Let D_t denote debt at the end of time period, t, for $t = 1, ..., T$, where D_0 is the debt inherited from the past. If r_t is the domestic interest rate at time t, equivalent to the government bond rate, then the debt servicing cost at time t, denoted d_t, is given by $d_t = r_t D_{t-1}$. The interest rate depends on the world interest rate, r_w, which is assumed to be constant, and a risk premium, $r_{p,t}$,so that $r_t = r_w + r_{p,t}$. Government expenditure includes welfare spending, W_t, which consists of untaxed transfer payments of $W_{B,t}$, received by non-pensioners, and aggregate (untaxed) superannuation benefits of $W_{S,t}$.[17] The levels per person are denoted $W^*_{S,t}$ and $W^*_{B,t}$, so that if $N_{S,t}$ and $N_{B,t}$ denote the number in receipt of the pension and welfare benefits respectively, $W_{S,t} = N_{S,t} W^*_{S,t}$ and $W_{B,t} = N_{B,t} W^*_{B,t}$.

All other spending at t is denoted by E_t. This is composed of spending on publicly-provided goods such as health and education, $E_{I,t}$, and other expenditure, $E_{O,t}$, so that $E_t = E_{I,t} + E_{O,t}$. The former may be considered as investment in human capital, while the other expenditure has no direct impact on individuals. As explained below, $E_{O,t}$ is assumed to have no direct impact on the labour supply, and thus incomes, of individuals. While $E_{I,t}$ does not have a direct impact, it influences income via its effect on productivity growth. Variations in these spending categories are produced by variations in per capita amounts, $E^*_{I,t}$ and $E^*_{O,t}$ and variations in the total population, N_t: hence $E_t = N_t \left(E^*_{I,t} + E^*_{O,t} \right)$.

Total government expenditure, G_t, is thus:

$$G_t = W_t + E_t + r_t D_{t-1} \tag{A.1}$$

Define R_t as total tax revenue from direct and indirect taxes, so that debt in period t is:

$$D_t = D_{t-1} + G_t - R_t \tag{A.2}$$

[17] In practice New Zealand Superannuation is taxable. However, this is allowed for in the calibration of the model, discussed below.

2 Income

Define $Y_{P,t}$ as total 'potential income' in period t, from labour and rental income. To allow for productivity growth at the rate ρ_t, write:

$$Y_{P,t} = (1 + \rho_t)\, Y_{P,t-1} \tag{A.3}$$

Let L_t indicate the ratio of actual to potential income, so that aggregate income can be written as $Y_t = L_t Y_{P,t}$.

Assume that all forms of income are taxed at the same rate. Then if S_t denotes aggregate savings at time, t, as defined above, these are all assumed to be invested at the going rate, r_t. Capital, K_t, is thus $K_t = K_{t-1} + S_{t-1}$. As this refers to the accumulation of savings, no depreciation is applied. As discussed above, the production side of the economy, including investment and capital accumulation, is not modelled explicitly. Hence aggregate income is:

$$Y_{A,t} = Y_t + r_t K_{t-1} \tag{A.4}$$

For simplicity, this assumes that the borrowing and lending rates are equal, and the same both for the government and individuals. The above specification does not allow for population growth. A simple adjustment is made by raising $Y_{A,t}$ by a proportion that depends on the growth rate, from period $t-1$ to t, of the population above working age.

3 Tax Revenue

Suppose that income tax is simply a constant proportion, τ_t, of taxable income. Revenue is also obtained from indirect taxes. Define V_t as indirect tax revenue at t, from a GST/VAT type of system, where v_t is the tax-exclusive rate applied to all expenditure. However, indirect taxes applied to E_t are ignored here since these are netted out in the government's budget constraint. The tax-inclusive indirect tax rate is $v_t/\,(1 + v_t)$.

Savings, S_t, are made from net income. Assume that all transfer payments, W_t, are consumed. Then if savings are a constant proportion, s_t, of post-tax income, $S_t = s_t\,(1 - \tau_t)\, Y_{A,t}$ and indirect tax revenue is:

$$V_t = \left[\frac{v_t\,(1 - s_t)\,(1 - \tau_t)}{1 + v_t}\right] Y_{A,t} + \left[\frac{v_t}{1 + v_t}\right] W_t \tag{A.5}$$

Total tax revenue, R_t, consists of income tax, plus V_t, plus other revenue, $R_{O,t}$. The latter is specified as an amount per capita, $R^*_{O,t}$, which is subject to an exogenous growth rate, along with growth arising from the increase each period in the population above working age. Substituting for V_t in $R_t = \tau_t Y_{A,t} + V_t + R_{O,t}$ gives:

$$R_t = \tau_t^* Y_{A,t} + \left[\frac{v_t}{1+v_t}\right] W_t + R_{O,t} \tag{A.6}$$

where τ_t^* is the overall effective income tax rate, given by:

$$\tau_t^* = \tau_t + v_t \frac{(1-s_t)(1-\tau_t)}{(1+v_t)} \tag{A.7}$$

Hence τ_t^* reflects the combined effect of the income and consumption tax rates.

4 Feedback Effects

The risk premium at time t is considered to be a function of $D_{t-1}/Y_{A,t-1} = DR_{t-1}$. Evidence suggests that the risk premium increases only slowly for relatively small values of this ratio, but increases rapidly once it exceeds a value of DR^*.[18] For $DR_{t-1} > DR^*$, suppose:

$$r_{p,t} = \theta_1 + \theta_2 DR_t + \theta_3 (DR_t)^2 \tag{A.8}$$

and for $DR_{t-1} \leq DR^*$ the premium increases linearly:

$$r_{p,t} = \theta_1 + \theta_2 DR^* + \theta_3 (DR^*)^2 - \theta_0 (DR^* - DR_{t-1}) \tag{A.9}$$

Suppose the saving rate, s_t, depends on the interest rate. In principle this effect is ambiguous, but it is assumed here that the interest-elasticity of savings is positive. This is reflected in a reduced-form relationship:

$$s_t = \theta_{11} + \theta_{12} r_t \tag{A.10}$$

with $\theta_{12} > 0$. Furthermore, the savings rate enters into the determination of the effective tax rate, τ_t^*, as shown in (A.7).

To capture adverse incentive effects of the tax and transfer system, suppose the variable, L_t, is a function of the tax rate, so that $L_t = L(\tau_t^*)$, with $dL_t/d\tau_t^* < 0$. Suppose the elasticity of taxable income, defined with respect to the effective net-of-tax rate, $1 - \tau_t^*$, is constant. Then:

$$L(\tau_t^*) = \theta_8 (1 - \tau_t^*)^{\theta_9} \tag{A.11}$$

[18] For example, see Ostry *et al.* (2010). The response of the risk premium to debt ratios is also discussed by Fookes (2011) and Lees (2013) in the New Zealand context.

Investments in the quality of human capital through both health and education enhance productivity. Suppose changes in ρ depends on previous growth of the per capita public expenditure component, $E^*_{I,t}$. The change in ρ depends on the change ℓ years previously, that is in $E^*_{I,t-\ell}$. If a dot above a variable indicates a proportionate change:

$$\dot{\rho}_t = \frac{\theta_4}{1 + \theta_5 \theta_6^{\dot{E}^*_{I,t-\ell}}} \tag{A.12}$$

This logistic form captures decreasing returns. If ρ_B is a 'base level' of productivity change:

$$\rho_t = \rho_B \left(1 + \dot{\rho}_t\right) \tag{A.13}$$

5 Details of Model Calibration

Despite the simplicity of the model, suitable orders of magnitude of many of the variables can be obtained from National Income data and demographic projections. The data sources and values are set out in detail in Creedy and Scobie (2015). Parameter values used for the various functions are listed in Table 4.

Table 4: Benchmark Parameter Values

Risk premium: For $DR_{t-1} > DR^*$, $r_{p,t} = \theta_1 + \theta_2 DR_{t-1} + \theta_3 \left(DR_{t-1}\right)^2$	
For $DR_{t-1} \leq DR^*$, $r_{p,t} = \theta_1 + \theta_2 DR^* + \theta_3 \left(DR^*\right)^2 - \theta_0 \left(DR^* - DR_{t-1}\right)$	
θ_0	0.026
θ_1	-0.03
θ_2	0.015
θ_3	0.0015
DR^*	1.0
Productivity growth changes: $\dot{\rho}_t = \theta_4 / \left(1 + \theta_5 \theta_6^{\dot{E}^*_{I,t-\ell}}\right)$	
θ_4	0.6
θ_5	35
θ_6	0.00005
ρ_B	0.015
ℓ	5
Incentive effects of taxation: $L\left(\tau_t\right) = \theta_8 \left(1 - \tau^*_t\right)^{\theta_9}$	
θ_8	1.0
θ_9	0.5
Saving rate: $s_t = \theta_{11} + \theta_{12} r_t$	
θ_{11}	0.03
θ_{12}	0.0833

Appendix B: A Productivity Change

Creedy and Scobie (2015) found that increasing the basic rate from 1.5 per cent per year to 1.94 per cent, the deterministic model resulted in a debt ratio of 20 per cent of GDP in 2053. The effect of changing the base level in the stochastic model is shown in Figure 13. This involves adjusting the productivity growth rates over the period 1973 to 2013 so that their geometric mean is equal to 1.94. The probabilities of exceeding specified debt ratios are in Figure 14.

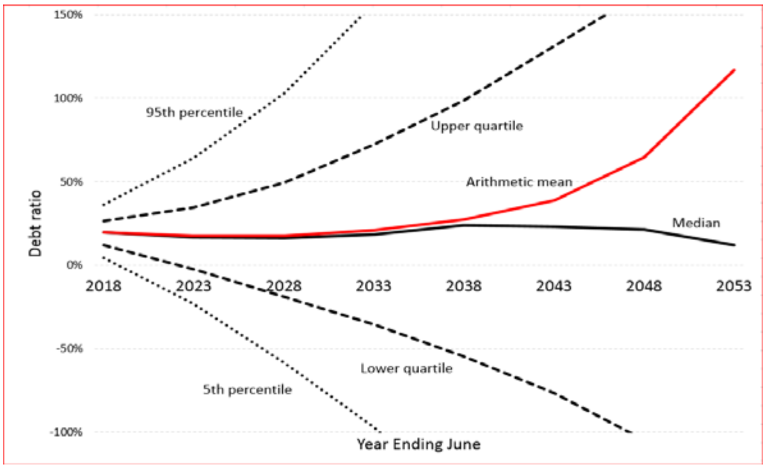

Figure 13: Stochastic Projections with Higher Base Productivity Growth Rate of 1.94 Per Cent

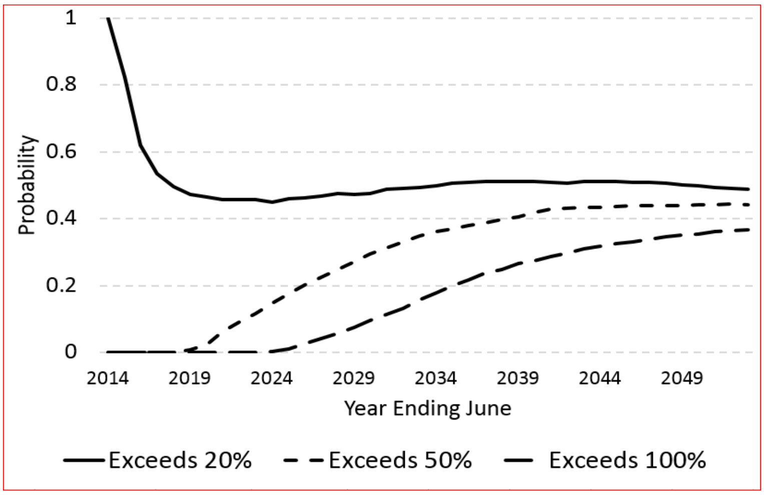

Figure 14: Probability of Exceeding Debt Ratios with Higher Base Productivity Growth

References

[1] Auerbach, A.J. and Hassett, K. (1998) Uncertainty and the design of long-run fiscal policy. *University of California Burch Working Paper*, no. B98-01.

[2] Auerbach, A.J. and Hassett, K. (2002) Uncertainty and the design of long-run fiscal policy. In *Demographic Change and Fiscal Policy* (ed. by Auerbach, A. and Lee, R.), pp. 73-92. Cambridge: Cambridge University Press.

[3] Armstrong, A. Draper, N., Nibbelink, A. and Westerhout, E. (2007) Fiscal prefunding in response to demographic uncertainty. *CPB Discussion Paper*, no. 85.

[4] Ball, C. and Creedy, J. (2014) Tax policy with uncertain future costs. *New Zealand Economic Papers*, 48, pp. 240-253.

[5] Barker, F.C., Buckle, R.A. and St Clair, R.W. (2008) Roles of fiscal policy in New Zealand. *New Zealand Treasury Working Paper*, WP 08/02.

[6] Barro, R.J. (1979) On the determination of the public debt. *Journal of Political Economy*, 87, pp. 940-971.

[7] Bell, M. and Rodway, P. (2014) Treasury's 2013 long-term fiscal statement: Assumptions and projections. *New Zealand Economic Papers*, 48, pp. 139-152.

[8] Bloom, D.E., Canning, D and Sevilla, J. (2001) The Effect of Health on Economic Growth: Theory and Evidence. *National Bureau of Economic Research Working Paper*, 8587. Available at: http://www.nber.org/papers/w8587.pdf

[9] Bloom, D.E. and Canning, D. (2003) Health as Human Capital and its Impact on Economic Performance. *The Geneva Papers on Risk and Insurance*, 28, pp. 304-315.

[10] Buckle, R.A. and Cruickshank, A.A. (2014) The requirements for fiscal sustainability in New Zealand. *New Zealand Economic Papers*, 48, pp. 111-128.

[11] Creedy, J. and Scobie, G.M. (2005) Population Ageing and Social Expenditure in New Zealand. *Australian Economic Review*, 38, pp. 19-39.

[12] Creedy, J. and Scobie, G.M. (2015) Debt Projections and Fiscal Sustainability with Feedback Effects. *New Zealand Treasury Working Paper*, no. 15/11.

[13] Creedy, J. and Makale, K. (2014) Social expenditure in New Zealand: Stochastic projections. *New Zealand Economic Papers*, 48, pp. 196-208.

[14] Davis, N. and Fabling, R. (2002) Population ageing and the efficiency of fiscal policy in New Zealand. *New Zealand Treasury Working Paper*, 02/11.

[15] Dixit, A.K. and Pindyck, R.S. (1994) *Investment under Uncertainty.* New York: Princeton University Press.

[16] Earle, David (2010) Tertiary education, skills and productivity. *Ministry of Education Tertiary Education Occasional Paper*, 2010/01. Available at: http://www.educationcounts.govt.nz/publications/series/ALL/tertiary-education,-skills-and-productivity/key-findings

[17] European Commission (2006) *The Long-Term Sustainability of Public Finances in the European Union.* European Economy, no. 4/2006.

[18] Fookes, C. (2011) Modelling shocks to New Zealand's fiscal position. *New Zealand Treasury Working Paper*, WP 11/02.

[19] Lassila, J. and Valkonen, T. (2004) Pre-funding expenditure on health and long-term care under demographic uncertainty. *The Geneva Papers on Risk and Insurance*, 29, pp. 620-639.

[20] Pindyck, R.S. (2008) Sunk costs and real options in antitrust analysis. *Issues in Competition Law and Policy*, pp. 619-640.

[21] Provost, L. (3013) Commentary on Affording Our Future: Statement on New Zealand's Long-term Fiscal Position. Available at: http://www.oag.govt.nz/2013/long-term-fiscal-position.

[22] Razzak, W.A. and Timmins, J. (2010) Education and labour productivity in New Zealand. *Applied Economics Letters* 17(2):169-173.

[23] Ter-Minassian, T. (2014) External Review of the Treasury's Fiscal Policy Advice. Available at: http://www.treasury.govt.nz/publications/informationreleases/fiscalpolicyadvice/pdfs/tfp
2908566.pdf

[24] The Treasury (2006) New Zealand's Long-term Fiscal Position (Wellington, New Zealand).

[25] The Treasury (2009) Challenges and Choices: New Zealand's Long-term Fiscal Statement (Wellington, New Zealand).

[26] The Treasury (2013a) Affording Our Future – Statement of New Zealand's Long-term Fiscal Position (Wellington, New Zealand).

[27] The Treasury (2013b) The Education Sector over the Long-term. Background paper prepared for the Long-term Fiscal Statement 2013. http://www.treasury.govt.nz/government/longterm/fiscalposition/2013/pdfs/ltfs-13-bg-eslt.pdf (Wellington, New Zealand).

[28] The Treasury (2013c) Long-term Fiscal Model for the Statement of the Long-term Fiscal Position 2013. (Wellington, New Zealand). http://www.treasury.govt.nz/government/longterm/fiscalmodel.

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