CAPITAL GAINS AND THE CAPITAL ASSET PRICING MODEL

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Martin Lally
School of Economics and Finance
Faculty of Commerce and Administration
Victoria University of Wellington

Tony van Zijl
School of Accounting and Commercial Law
Faculty of Commerce and Administration
Victoria University of Wellington

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Please address all correspondence to:
Professor Tony van Zijl
School of Accounting and Commercial Law
Victoria University of Wellington
PO Box 600, Wellington, New Zealand
Phone: +64-4-463-5329   Fax: +64-4-463-5076
Email: tony.vanzijl@vuw.ac.nz
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Abstract

This paper shows that, in the presence of differential taxation of ordinary income and capital gains, use of the Officer (1994) version of the Capital Asset Pricing Model can result in significant misestimation of the cost of equity capital. In particular, with a high dividend yield, the cost of equity may be underestimated by four percentage points. Underestimation is of particular significance in the context of setting output prices for regulated utility firms.

JEL: G12, G31.

Key Words: Capital Asset Pricing Model; Personal Taxes.

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1. INTRODUCTION

The Officer (1994) approach to estimation of the expected cost of equity capital under a system of dividend tax imputation, has been widely accepted in Australia among both the academic and practitioner communities in finance.

Officer views the tax effect of imputation as a company tax phenomenon and measured rates of return thus incorporate the effect of tax credits. He states the CAPM as:

\[
E(\hat{R}_j) = R_F + \left[ E(\hat{R}_m) - R_F \right] \beta_j
\]

where

- \( R_F \) = risk free rate for the period
- \( \hat{R}_j \) = rate of return on the equity of company \( j \), including imputation credits
- \( \hat{R}_m \) = rate of return on the market portfolio, including imputation credits
- \( \beta_j \) = \( \frac{Cov(\hat{R}_j, \hat{R}_m)}{Var(\hat{R}_m)} \)

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1 A dividend imputation tax system was adopted in the UK in 1973, in Australia in 1987, and New Zealand followed in 1988. The basic system now exists in a wide range of countries; Smith (1993) provides a comprehensive review of the range of models across countries.

2 Examples of textbooks that have adopted the Officer approach are Bishop et al. (1993, Ch. 16) and Peirson et al (1998, Ch. 15). The reports of the utility regulatory authorities in Victoria and New South Wales show general acceptance of the Officer approach for utility rate setting not only by the authorities but also by the entities affected and by their financial advisors. Anecdotal evidence the authors are aware of and their own casual observation both support there being a similar degree of acceptance of the approach for applications in the private sector.

3 Officer’s view of imputation tax stands in contrast to the view of imputation tax being an investor tax phenomenon. Under the latter view, imputation is seen as providing a reduced tax rate for investors on income from dividends. The views differ only by “a transform” and therefore the difference is not substantive. The investor tax view of imputation tax is the view that has been taken almost uniformly in the literature on the CAPM and imputation tax systems. The literature effectively starts with Brennan (1970) and includes Stapleton and Burke (1977), Ashton (1989, 1991), Cliffe and Marsden (1992), Lally (1992), Okunev and Tahir (1992), Monkhouse (1993), van Zijl (1993), Brailsford and Davis (1995), Dempsey (1996), and Brailsford and Heaney (1998). The general body of this literature is reviewed in Lally (2000).
The apparently simple form of Officer’s CAPM is due to the implicit assumption that
the tax rates on capital gains and ordinary income are equal (or, more generally, that the
weighted average of the tax ratio [ ] in equation (5) below is zero). In Australia, the
effective capital gains tax rate has been and will remain less than the tax rate on ordinary
income because (i) in the past capital gains tax has applied only to the real element of a
gain, and in future individuals and superannuation funds will pay the tax on only 50%
and 67% respectively of a gain, and (ii) the tax is payable only on realisation of a gain
and can therefore be deferred. The assumption of equality of the rates on capital gains
and ordinary income is thus a significant abstraction from the reality of the Australian
tax system and use of the Officer CAPM for estimation of the cost of capital therefore
inevitably involves a degree of approximation.\footnote{The authors’ experience of cost of capital estimation is that practitioners in New Zealand have not followed the Officer approach. Instead, imputation is viewed as an investor tax phenomenon and measured rates of return therefore do not reflect tax credits. The form of the CAPM most commonly used has been:}

$$E(R_j) = R_F (1 - T_m) + [E(R_m) - R_F (1 - T_m)] \beta_j$$

where $R_j$ is the return on $x$ excluding tax credits, and $T_m$ is the statutory maximum personal tax rate which
until recently was equal to the corporate tax rate, 0.33. The model is often referred to as the “after tax”
form of the CAPM. It assumes that the tax on capital gains is zero across all investors, and that there is full
attachment of imputation credits. There is evidence that, in recent times, some practitioners have switched
to versions of the CAPM which give recognition to the fact that the capital gains tax, while low on average,
is not zero. Examples of such models are Lally (1992) and van Zijl (1993). It is not apparent why practice
in New Zealand has developed differently from that in Australia. The two countries have extensive
commercial links and the switch from the classical to the imputation tax systems occurred at about the same
time. The assumption of a zero capital gains tax underlying the New Zealand approach is unrealistic but
probably less so than the assumption made in Australia of equality of the tax rates across capital gains and
ordinary income. On the other hand, the New Zealand approach of assuming full attachment of imputation
credits is unrealistic, whereas the Australian approach has made allowance for this.
degree have potentially significant implications for project adoption and for the setting of output prices for regulated utilities. Accordingly the adoption of a CAPM that recognises both dividend imputation and differential taxation of ordinary income and capital gains would seem to be justified.\textsuperscript{5}

2. THE CAPM WITH DIFFERENTIAL TAXATION

Under the Officer approach, the rate of return on a stock comprises the capital gain, the cash dividend, and the imputation credits (so far as they can be used). Thus the rate of return on the shares of company $j$, denoted by $\hat{R}_j$, is\textsuperscript{6}

$$\hat{R}_j = \frac{P_{j1} - P_{j0}}{P_{j0}} + \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right)$$

where

$D_j =$ company $j$'s cash dividend per share over the next period

$P_{jt} =$ price of shares in company $j$ at time $t$

$IC_j =$ imputation credits attached to the cash dividend of company $j$

$U =$ market wide utilisation rate for imputation credits, which ranges from 0 to 1

With returns defined in this way, Officer’s form of the CAPM is

$$E(\hat{R}_j) = R_F + [E(\hat{R}_m) - R_F] \beta_j$$  \hspace{1cm} (1)

However, as noted above, this form is based on the assumption of equal tax rates across capital gains and ordinary income. The model developed in this paper does not assume equality of the tax rates; the derivation is set out below and follows Elton and Gruber (1984).

\textsuperscript{5} The form of the CAPM adopted could be the model derived in this paper (and thus continue with the company tax view) or alternatively one of the models referred to in footnote 3, based on the investor tax view.
Investor $i$ chooses an “efficient” portfolio by combining the riskless asset with the “tangency” portfolio $K$. Then from Roll (1977) it follows that with unrestricted short selling, the after tax expected return on asset $j$ to investor $i$, denoted by $E(\hat{r}_{ji})$, is related to the beta of $j$ against portfolio $K$, as:

$$E(\hat{r}_{ji}) = r_{Fi} + \left[ E(\hat{r}_{Ki}) - r_{Fi} \right] \frac{Cov(\hat{r}_{ji}, \hat{r}_{Ki})}{Var(\hat{r}_{Ki})}$$

where $\hat{r}_{Ki}$ is investor $i$’s after tax return on portfolio $K$ and $r_{Fi}$ is the investor’s after-tax return on the riskfree asset.

Defining $T_{gi}$ as investor $i$’s tax rate on capital gains and $T_{pi}$ as the investor’s tax rate on ordinary income, it follows that:

$$\hat{r}_{ji} = \frac{P_{j1} - P_{j0}}{P_{j0}} (1 - T_{gi}) + \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right) (1 - T_{pi})$$

$$= \left\{ \frac{P_{j1} - P_{j0}}{P_{j0}} + \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right) \right\} (1 - T_{gi}) - \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right) (T_{pi} - T_{gi})$$

$$= \hat{R}_j (1 - T_{gi}) - \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right) (T_{pi} - T_{gi})$$

and, similarly:

$$\hat{r}_{Ki} = \hat{R}_K (1 - T_{gi}) - \frac{D_k}{P_{K0}} \left( 1 + U \frac{IC_k}{D_k} \right) (T_{pi} - T_{gi})$$

Also

$$r_{Fi} = R_F (1 - T_{pi})$$

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6 In this formulation the term $U(\text{IC}_j)/P_{j0}$ is equivalent to Officer’s term $\tau_j$, defined as the value of the tax credits expressed as a proportion of the current value of the share (Officer, 1994, p. 7).
Then, assuming that the end of period dividend is non-stochastic, substituting these results into (2) and dividing by $(1-T_{gi})$ gives:

$$E(\hat{R}_j) - R_F \left[ \frac{1-T_{pi}}{1-T_{gi}} \right] = \frac{E(\hat{r}_{Ki}) - r_{Fi}}{\text{Var}(\hat{r}_{Ki})} (1-T_{gi}) \text{Cov}(\hat{R}_j, \hat{R}_K) + \frac{D_j}{P_{j0}} \left[ \frac{T_{pi}-T_{gi}}{1-T_{gi}} \right] \left( 1 + U \frac{IC_j}{D_j} \right)$$

(3)

Defining $w_i$ as the fraction of aggregate market investments held by investor $i$, and $\lambda_i$ as

$$\lambda_i = \frac{E(\hat{r}_{Ki}) - r_{Fi}}{\text{Var}(\hat{r}_{Ki})} (1-T_{gi})$$

then, multiplying (3) through by $w_i$, dividing by $\lambda_i$, summing across all investors, and noting that $\sum w_i \hat{R}_K = \hat{R}_m$, gives:

$$\left( \sum \frac{w_i}{\lambda_i} \right) E(\hat{R}_j) - R_F \sum \frac{1-T_{pi}}{1-T_{gi}} \frac{w_i}{\lambda_i} = \text{Cov}(\hat{R}_j, \hat{R}_m) + \frac{D_j}{P_{j0}} \left( 1 + U \frac{IC_j}{D_j} \right) \sum \frac{T_{pi}-T_{gi}}{1-T_{gi}} \frac{w_i}{\lambda_i}$$

that is,

$$E(\hat{R}_j) - R_F (1-T) = \frac{\text{Cov}(\hat{R}_j, \hat{R}_m)}{\sum \frac{w_i}{\lambda_i}} + d_j \left( 1 + U \frac{IC_j}{D_j} \right) T$$

(4)

where

$$d_j = \frac{D_j}{P_{j0}}$$

$$T = \sum \left[ \frac{T_{pi}-T_{gi}}{1-T_{gi}} \right] x_i$$

(5)

$$x_i = \frac{w_i}{\lambda_i} = \sum \frac{w_i}{\lambda_i} = \frac{w_i}{\lambda_i} \cdot \sum \frac{w_i}{\lambda_i} = \frac{\frac{w_i}{\lambda_i}}{\sum \frac{w_i}{\lambda_i}} \sum \frac{E(\hat{r}_{Ki}) - r_{Fi}}{\text{Var}(\hat{r}_{Ki})} (1-T_{gi})$$

(6)

Since (4) holds for all securities, it must also hold for the market, that is:

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7 In respect of the assumption of non-stochastic dividends, Lally (1999) shows that, if dividends are instead assumed to be stochastic, the only material effect is to replace dividends by their expectations in the resulting model.
\[ E(\hat{R}_m) - R_F (1-T) = \frac{\text{Var}(\hat{R}_m)}{\sum \frac{w_i}{\lambda_i}} + d_m \left( 1 + U \frac{IC_m}{D_m} \right) \]

Solving for \( \sum (w_i / \lambda_i) \), and substituting into (4), gives the CAPM with differential taxation:

\[ E(\hat{R}_j) = R_F (1-T) + d_j \left( 1 + U \frac{IC_j}{D_j} \right) T + \left[ E(\hat{R}_m) - R_F (1-T) - d_m \left( 1 + U \frac{IC_m}{D_m} \right) T \right] \beta_j \]

that is:

\[ E(\hat{R}_j) = R_F + [E(\hat{R}_m) - R_F] \beta_j + \Delta_j \]  \hspace{1cm} (7)

where

\[ \Delta_j = T \left[ d_j \left( 1 + U \frac{IC_j}{D_j} \right) - R_F + \beta_j \left[ R_F - d_m \left( 1 + U \frac{IC_m}{D_m} \right) \right] \right] \] \hspace{1cm} (8)

Comparison of the CAPM derived above, with the Officer CAPM (equations (7) and (1) respectively), shows that the approximation made in using the Officer CAPM is given by \( \Delta_j \).

From (8) it is readily seen that (in general) \( \Delta_j = 0 \) if

1. \( T = 0 \), that is, that capital gains and ordinary income are taxed equally, on average. A sufficient condition is that \( T_{gi} = T_{pi} \) for all \( i \),

or
2. \( \beta_j = 1 \), \( d_j = d_m \) and \( IC/D_j = IC_m/D_m \), that is, the stock matches the market in respect of each of beta, dividend yield, and imputation credits relative to dividends.

As already noted earlier, capital gains and ordinary income are not taxed equally under the Australian tax system and therefore condition (1) does not hold. Condition (2) must hold (in a loose way) “on average” and therefore the Officer CAPM will on average provide a very good approximation to the result of applying the CAPM with differential taxation. However, for any given security, the degree of approximation could be significant.
3. DEGREE OF THE APPROXIMATION

We examine the degree of approximation by substituting into the formula for $\Delta_j$ a plausible value or range of values for each of the parameters determining $\Delta_j$.

3.1 Parameter Values

The relevant parameters are the riskfree rate, beta, dividend yield, the ratio of imputation credits to cash dividends, the utilisation rate for imputation credits, and the tax parameter $T$.

Consistent with recent experience, we assume that the risk free rate is 0.065. In respect of beta, a plausible cross-sectional range is 0.5 to 1.5. The market dividend yield is about 0.03 (data courtesy of Ord Minnett) and a plausible cross-sectional range is 0 to 0.1. The market ratio of imputation credits to cash dividends is 0.33 (data courtesy of Ord Minnett). At the firm level this can range from 0 to a maximum of 0.56 (based on the current corporate tax rate of 36%). The maximum will fall to 0.49 during the transition (while the corporate tax rate is 33%), and to 0.43 when the corporate tax rate reduces to 30%. We therefore consider a cross-sectional range from 0 to 0.50.

The utilisation rate, $U$, is commonly estimated at 0.6, which is consistent with the fact that foreign investors are significant in the market but cannot use the credits (the supporting studies are Brown and Clarke, 1993; Bruckner et al, 1994; Officer and Hathaway, 1995). However, the CAPMs considered in this paper assume that national sharemarkets are fully segmented. Consequently the utilisation rate should be 1 other than for the market weight of Australian investors unable to use the credits. The only investors of this type are tax-exempt, and Wood (1997, footnote 10) estimates that their market weight is just 3-4%. Thus 1.0 is a reasonable estimate for $U$. However we also consider the impact of using instead the more common estimate of 0.6.

The final parameter requiring estimation is $T$. As shown in equation (5), this is a weighted average over investors of the tax ratio $\frac{\tau_j}{\tau_i}$, with weights $x_i$. Estimation of $T$ thus requires specification of the relevant investor set, estimation of their tax rates on both
ordinary income and capital gains, and estimation of the weights, \( x_i \). The holders of Australian equities can be classified as foreigners, companies, superannuation funds and individuals. Since the CAPM in question, along with the Officer version, assumes that national capital markets are segregated, then it would be inconsistent to recognise foreign investors. Accordingly we omit them from consideration. In respect of corporate holdings of shares in other companies, inclusion of them would lead to double-counting. Consequently we omit them. If companies were subject to taxation on the dividends received from other companies the personal tax rates faced by the ultimate recipients (individuals and superannuation funds) would need to be increased to reflect this. However, companies are not taxed on dividend income, and therefore this potential complication is absent. Thus, having excluded both foreign investors and corporate shareholders, only individuals and superannuation funds need to be considered.

Under the new tax system, the highest marginal tax on ordinary income for individuals will remain at 0.47 but many individuals will actually pay lower rates because of the progressive scale or because of income splitting. We assume that individuals are, on average, subject to a 0.35 tax rate on ordinary income. In contrast, we assume that superannuation funds will face the statutory tax rate of 0.15 on ordinary income.

Turning to capital gains, in Australia there are two reasons for taxes on capital gains being lower than on ordinary income. Firstly, only part of the assessable gain is taxable under the new tax system. Individuals will be subject to tax on only 50% of assessable capital gains, and superannuation funds on 67% of assessable capital gains. Under the old tax system, only the real gain was subject to tax. Secondly, capital gains are taxed only on realisation and the resulting opportunity to defer the tax effectively reduces the rate of the tax. Protopapadakis (1983) estimates that the opportunity to defer reduces the tax rate on capital gains by about 50\%.\(^8\) These two features of the taxation regime for capital gains suggest that on average individual investors and superannuation funds will pay capital gains tax at only 25\% and 33\% respectively of the rates applicable to ordinary income. Applied to the above estimates for tax rates on ordinary income this

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\(^8\) The opportunity to defer lowers the effective tax rate not only because of the time value of money but also, as Hamson and Ziegler (1990, p. 49) note, because gains can be realised when the investor’s tax rate is lower, such as in retirement.
implies effective capital gains tax rates of 0.0875 and 0.05 respectively. That capital gains are taxed less onerously than ordinary income, because of exemptions and/or the deferral option, is well recognised, not only for Australia (see Howard and Brown, 1992) but other countries such as the US (see Constantinides, 1984) and the UK (see Ashton, 1991).

In respect of the weights applied to the ratio in equation (5), the weight for investor \( i \) is

\[
x_i = \frac{w_i}{\sum w_i} \left[ \frac{E(\hat{r}_{Ki}) - r_{Fi}}{\text{Var}(\hat{r}_{Ki})} \right] \left( 1 - T_{gi} \right)
\]

The term \([\ ]\) reflects the risk aversion of investor \( i \), and is unknown. Given the absence of information about cross-sectional variation in risk aversion, we assume that it is uniform across investor tax categories. The weight for investor \( i \) thus reduces to

\[
x_i = \frac{w_i}{\left( 1 - T_{gi} \right)} \left( 1 - T_{gi} \right)
\]

For individuals and superannuation funds the market investment weights, \( w_i \), are 23% and 11% respectively.\(^9\) Substituting for these values and the capital gains tax rates estimated above yields values for \( x_i \) of 0.68 for individuals and 0.32 for superannuation funds. Substitution into equation (5) produces an estimate for the tax parameter \( T \) of 0.23.

The results of research on the behaviour of ex-day returns provides broad support for the estimate for \( T \).\(^{10}\) Based on Australian data for the period 1986-1995, Hathaway and Officer (1995, Table 2) report an intercept of 0.70 to 0.84 in regressions of \( \Delta P/D \) (share price change over the ex-dividend day divided by the cash dividend) on the imputation credits attached to the dividend. The results are consistent with the earlier work by Brown and Walter (1986), covering the period 1974-1985. The intercept in such

\(^9\) These weights are taken from the ASX Fact Book, 1999. We are grateful to David McCallum of ABN AMRO for advice on estimation of the average tax rates.

\(^{10}\) We are grateful to John Redmayne of PricewaterhouseCoopers for suggesting this approach to estimating \( T \).
regressions can be interpreted as being the mean value of $\Delta P/D$ in the absence of imputation credits. In such circumstances, arbitrage suggests that

$$\Delta P(1-T_{ga}) = D(1-T_{pa})$$

where $T_{ga}$ is the average tax rate on capital gains and $T_{pa}$ is the average rate on ordinary income. Thus

$$\frac{\Delta P}{D} = \frac{(1-T_{pa})}{(1-T_{ga})} = 1 - \frac{T_{pa} - T_{ga}}{1 - T_{ga}} = 1 - T_a$$

The intercept values of .70 to .84 thus suggest an estimate for $T_a$ in the range 0.16 to 0.30. In general $T_a$ is not equal to $T$, because of variation in the capital gains tax rates across investors. However, in this case, there is little such variation and therefore $T_a$ should be close to $T$. The research into ex-day returns thus indicates that $T$ lies in the range of 0.16 to 0.30. This approach to estimating $T_a$ does give rise to a number of concerns, including statistical uncertainty in the estimate of the intercept and various alternative explanations for intercept values differing from 1. The latter include microstructure explanations (Frank and Jagannathan, 1998), evidence of anomalous behaviour in the broader period around the ex-day (Brown and Walter, 1986) and the possibility that the value reflects the actions of arbitrageurs in a particular tax bracket buying just before and selling just after the ex-day. In view of these concerns, in the sensitivity analysis we adopt the estimate for $T$ of 0.23 derived above but, in recognition of the uncertainty surrounding that estimate, we consider a range of possible values from 0.13 to 0.33.

3.2 Sensitivity Analysis

Table 1 below shows the values of $\Delta_j$ for selected combinations of the values of the parameters. Adopting the estimates $T = 0.23$, $R_F = 0.065$, $U = 1.0$, $d_m = 0.03$, and $IC_m/D_m = 0.33$, the sensitivity of $\Delta_j$ to variations in $d_j$, $IC_j/D_j$, and $\beta_j$ is examined by varying one of the latter three parameters over its relevant range and holding the other two at their boundary values. Thus varying $d_j$ over the interval [0, 0.10] results in $\Delta_j$ varying as follows:
over \([-0.012, 0.011]\) for \(\text{IC}_j/D_j = 0\) and \(\beta_j = 0.5\)
over \([-0.006, 0.017]\) for \(\text{IC}_j/D_j = 0\) and \(\beta_j = 1.5\)
over \([-0.012, 0.022]\) for \(\text{IC}_j/D_j = 0.5\) and \(\beta_j = 0.5\), and
over \([-0.005, 0.028]\) for \(\text{IC}_j/D_j = 0.5\) and \(\beta_j = 1.5\)

Similarly, varying \(\text{IC}_j/D_j\) over the interval \([0, 0.5]\) results in \(\Delta_j\) varying as follows:
over \([-0.012, -0.012]\) for \(d_j = 0\) and \(\beta_j = 0.5\)
over \([-0.006, -0.005]\) for \(d_j = 0\) and \(\beta_j = 1.5\)
over \([0.011, 0.022]\) for \(d_j = 0.1\) and \(\beta_j = 0.5\), and
over \([0.017, 0.028]\) for \(d_j = 0.1\) and \(\beta_j = 1.5\)

Finally, varying \(\beta_j\) over the interval \([0.5, 1.5]\) results in \(\Delta_j\) varying as follows:
over \([-0.012, -0.006]\) for \(d_j = 0\) and \(\text{IC}_j/D_j = 0\)
over \([-0.012, -0.005]\) for \(d_j = 0\) and \(\text{IC}_j/D_j = 0.5\)
over \([0.011, 0.017]\) for \(d_j = 0.1\) and \(\text{IC}_j/D_j = 0\), and
over \([0.022, 0.028]\) for \(d_j = 0.1\) and \(\text{IC}_j/D_j = 0.5\)

The sensitivity of \(\Delta_j\) to variation in \(U\) is examined by repeating the above calculations with \(U = 0.6\) instead of 1.0. The results are shown in the sixth column of Table 1.

Finally, we consider \(T\). Since \(T\) is proportional to \(\Delta_j\) then, if \(T\) increases by 43% from 0.23 to 0.33, the absolute value of \(\Delta_j\) also increases by 43%. Similarly, if \(T\) is reduced to .13, the absolute value of \(\Delta_j\) declines by 43%. This is evident from comparison of the fourth and seventh columns of Table 1.

The conclusions from this sensitivity analysis are as follows. First, across the range of values considered for the parameters, \(\Delta_j\) varies dramatically: with \(T = .23\), the variation is from \(-0.012\) to 0.028; with \(T = 0.33\), the variation increases proportionately from \(-0.017\) to 0.041. Second, variations in each of \(\beta_j, \text{IC}_j/D_j\) and \(U\) does not, in general, result in a significant effect on the value of \(\Delta_j\) (defined as more than 0.01). Third, variations in \(d_j\) are significant for any combination of values of the other parameters.
The key influences on $\Delta_j$ are thus $d_j$ and $T$. For companies with a high dividend yield, use of the Officer CAPM results in underestimation of the cost of equity capital. Conversely, for low dividend yields, the cost of equity capital will be overestimated. As $T$ increases this effect is magnified. The most significant underestimation result occurs with large values for both $d_j$ and $T$, where the Officer CAPM would underestimate the cost of equity capital by as much as 0.041.

3.3 Consequences

The consequences of the approximation identified in the preceding section are twofold. First, since the cost of equity is a component of the weighted average cost of capital, which is used to calculate the present value of the cash flows from prospective projects, use of the Officer CAPM could lead to the incorrect rejection and/or acceptance of some projects. Undoubtedly the adoption decision for many projects will not be sensitive to variations in the cost of equity to the degree shown here. Nevertheless the decision for at least some projects will be affected. For projects with very long lives, variations of even two percentage points to the discount rate can have a substantial effect upon present value, and therefore on the adoption decision.

The second consequence is for utility companies whose output prices are set on the basis of estimated cost of capital, such as those involved in electricity, gas or airports. The recent price determination for electricity companies issued by the regulator for Victoria (Office of the Regulator-General, Victoria, 2000) provides a relevant example. The parameter values adopted were $R_F = 0.062$, $U = 0.60$, and $\beta_j$ was set equal to 1.00 for all companies. Among the companies subject to the regulations is Envestra, which has a dividend yield of 0.097, and a ratio of imputation credits to cash dividends of zero (data from Bloomberg). Thus, using the additional market parameter estimates discussed above, of $T = 0.23$, $d_m = 0.03$ and $IC_m/D_m = 0.33$, the resulting value of $\Delta$ for the company is 0.014. That is, in this case, the Officer CAPM underestimates the cost of equity capital by 1.4 percentage points. If $T = 0.33$, this rises to two percentage points. The revenue implications of this underestimation would be very substantial for the

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12 With an average leverage level for Australian companies of 19% (Ernst and Young, 2000), the cost of equity is the dominant component in the weighted average cost of capital for the average Australian company.
company. Furthermore, this is not a pathological case as high dividend yields are common amongst utility companies.

One response to the analysis presented above might be to argue that the true value of $T$ is highly uncertain and that the bounds on the value of $\Delta$ are modest. Consequently a departure from the Officer model in favour of the model presented here is not justified. However, the Officer CAPM essentially represents a first step modification to the standard version of the CAPM (Sharpe, 1964; Lintner, 1965; Mossin, 1966), in that it incorporates dividend imputation but not differential taxation of capital gains and ordinary income. Defining asset $j$’s return in the standard way as

$$R_j = \frac{P_{j1} - P_{j0}}{P_{j0}} + \frac{D_j}{P_{j0}}$$

then the standard version of the CAPM states that

$$E(R_j) = R_F + \left[ E(R_m) - R_F \right] \beta_j$$

(9)

The Officer CAPM in equation (1) can be expressed as

$$E\left( R_j + d_j U \frac{IC_j}{D_j} \right) = R_F + \left[ E\left( R_m + d_m U \frac{IC_m}{D_m} \right) - R_F \right] \beta_j$$

that is

$$E(R_j) = R_F + \left[ E(R_m) - R_F \right] \beta_j + \theta_j$$

(10)

where

$$\theta_j = U \left[ \beta_j d_m \frac{IC_m}{D_m} - d_j \frac{IC_j}{D_j} \right]$$

(11)

13 Because returns are defined in this way the beta in the standard CAPM will be defined against such returns, and therefore might seem to differ from the beta in the Officer CAPM. However the Officer model (as with the model presented in this paper) arises by assuming, inter alia, that dividends are non-stochastic. Consequently the betas will be the same.
Comparison of (10) with (9) shows that the Officer CAPM specification of $E(R_j)$ differs from that of the standard version of the CAPM by the addition of $\theta_j$. Similarly, the CAPM derived in this paper, at (7) above, can be restated as

$$E(R_j) = R_F + [E(R_m) - R_F] \beta_j + \theta_j + \Delta_j$$

Thus, the standard version of the CAPM mis-estimates $E(R_j)$ by the amount $(\theta_j + \Delta_j)$, and the Officer model is a first step in correcting this through the addition of $\theta_j$.

Table 1 shows the values of $\theta_j$ and $(\theta_j + \Delta_j)$. Two significant conclusions are apparent from a comparison of $\Delta_j$, $\theta_j$, and $(\theta_j + \Delta_j)$, using the values for $U$ and $T$ of 1 and 0.23 respectively. First, across the eight cases considered in the rows of Table 1, the average absolute values for $\theta_j$ and $\Delta_j$ are comparable, at 1.7% and 1.4% respectively. Thus, looked at in isolation, the Officer adjustment to the standard CAPM ($\theta_j$) and the adjustment to the Officer model proposed in this paper ($\Delta_j$) are of comparable importance. Second, across the eight cases considered in Table 1, the average absolute error in the Officer model ($\Delta_j$) is much the same as that in the standard CAPM ($\theta_j + \Delta_j$). This is because $\Delta_j$ is typically opposite in sign to $\theta_j$. Consequently, as often as not, application of the Officer CAPM either adjusts the standard CAPM in the wrong direction (relative to the appropriate adjustment of $\theta_j + \Delta_j$) or adjusts in the appropriate direction by over twice the appropriate level. Thus, as often as not, the Officer CAPM produces a cost of equity further from the appropriate value than that produced by the standard CAPM. A dramatic example appears in the last row of Table 1: the standard model overstates the cost of equity by only 0.7 percentage points ($\theta_j + \Delta_j$) whereas the Officer model understates it by 2.8 percentage points ($\Delta_j$).

These points imply that, if dividend imputation is to be recognised, it is at least as important to additionally recognise differential personal taxation of capital gains and ordinary income. Such a CAPM could take the form derived in this paper (reflecting the

14 If $U = 0.6$, the figures become 1% and 1.3% respectively.
15 Examination of the formulas for $\Delta_j$ and $\theta_j$ in equations (8) and (11) respectively provides the explanation. The term $\Delta_j$ is a positive function of $d_j$ and $IC/D_j$, and a negative function of $d_m$ and $IC_m/D_m$. For $\theta_j$ the reverse holds.
company tax view of imputation) or, alternatively, a model reflecting the investor tax view of imputation.

4. CONCLUSION

In this paper we have shown that, with differential taxation of capital gains and ordinary income, the Officer CAPM may significantly misestimate the cost of equity capital. Relative to a model that additionally allows for this differential taxation, the degree of the approximation from use of the Officer CAPM is on average zero and sensitivity analysis shows that in many circumstances the degree of approximation is quite small. However, where the dividend yield is very high, the cost of equity could be underestimated by around four percentage points.

The principal consequences of this are twofold. First, some projects will be improperly accepted or rejected. Second, for companies whose output prices are set on the basis of cost of capital, such as utility companies, underestimation of the cost of equity capital could have significant revenue implications. Accordingly it would seem to be appropriate to switch to the use of a CAPM version that recognises both dividend imputation and differential taxation of capital gains and ordinary income. Such a CAPM could take the form derived in this paper (reflecting the company tax view of imputation) or, alternatively, a model reflecting the investor tax view of imputation.
REFERENCES


Ernst and Young, 2000, Country Leverages and their Relevance to the Valuation of New Zealand Companies, unpublished.


TABLE 1
The Degree of Approximation in the Officer Model

<table>
<thead>
<tr>
<th>$d_j$</th>
<th>$IC/D_j$</th>
<th>$\beta_j$</th>
<th>$\Delta_j$</th>
<th>$\theta_j$</th>
<th>$\Delta_j + \theta_j$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$T = .13$</td>
<td>$T = .23$</td>
<td>$T = .23$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$U = 1$</td>
<td>$U = 1$</td>
<td>$U = .6$</td>
</tr>
<tr>
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<td>-.012</td>
<td>-.012</td>
</tr>
<tr>
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<td>1.5</td>
<td>-.004</td>
<td>-.006</td>
<td>-.005</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>-.007</td>
<td>-.012</td>
<td>-.012</td>
</tr>
<tr>
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<td>0.5</td>
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<td>-.005</td>
<td>-.005</td>
</tr>
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<td>.011</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1.5</td>
<td>.016</td>
<td>.028</td>
<td>.025</td>
</tr>
</tbody>
</table>

This table shows values for $\Delta_j$, $\theta_j$ and their sum for various combinations of the firm-specific parameters $d_j$, $IC/D_j$ and $\beta_j$, and the market-wide parameters $T$ and $U$. All calculations assume $R_F = .065$, $d_m = .03$ and $IC_m/D_m = .33$. 