TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

Estimating bias of technical progress with a small dataset

Mohammed S Khaled

SEF WORKING PAPER 8/2017

The Working Paper series is published by the School of Economics and Finance to provide staff and research students the opportunity to expose their research to a wider audience. The opinions and views expressed in these papers are not necessarily reflective of views held by the school. Comments and feedback from readers would be welcomed by the author(s).

Further enquiries to:

The Administrator
School of Economics and Finance
Victoria University of Wellington
P O Box 600
Wellington 6140
New Zealand

Phone: +64 4 463 5353
Email: alice.fong@vuw.ac.nz

Working Paper 8/2017

ISSN 2230-259X (Print)

ISSN 2230-2603 (Online)

Estimating Bias of Technical Progress with a Small Dataset

Mohammed S. Khaled

School of Economics and Finance,
Victoria University of Wellington,
PO Box 600,
Wellington,
NEW ZEALAND.

Contact email: Mohammed.Khaled@vuw.ac.nz
Phone: +64 - 04 - 463 - 5787
Fax: +64 - 04 - 463 - 5014

ESTIMATING BIAS OF TECHNICAL PROGRESS WITH A SMALL DATA SET

Abstract:

Economic historians frequently face the challenge of estimation and inference when only a small sample of the relevant data is available. We illustrate solutions to the challenges through a case study analysis of the Uselding and Juba (1973) data. They have only seven observations available to estimate of the bias of technical progress in United States manufacturing in the nineteenth century. They are able to offer estimates of the bias only by assuming that production technology is not Cobb-Douglas, technical progress is non-neutral and that elasticity of substitution between labour and capital is less than 0.9. These assumptions could not be tested owing to the paucity of the required historical data. This case study illustrates the use of both additional theoretical information and appropriate statistical techniques to alleviate problems of estimation and inference with small samples.

JEL Classification:

Key Words:

Biased Technical Progress; Elasticity of Substitution; Translog Cost Function; Total Factor Productivity; Bootstrapping.

1. Introduction

American manufacturing in the nineteenth century is characterized by wages rising faster than rental prices of capital, and adoption of increasingly capital intensive methods of production. An obvious reason for such a non-neutral change in factor usage is the so called 'substitution effect', i.e., a greater use of the relatively cheaper input even when output does not change. Biased factor growth can result also from non-homothetic scale effects, i.e., from moving along a non-linear expansion path as output grows. Such changes in factor ratios at a point of time are of course limited to the currently known set of techniques of production.

Yet another source of bias, which has attracted the most attention, is technical progress. Its impact is realised only with the passage of time. Rothbarth (1946) put forward the hypothesis that labour scarcity, and hence increasing costs of labour, imparted a labour saving bias to technical progress in US manufacturing during the nineteenth century. Rising wages tend to erode the share of capital in total incomes; owners of capital try to protect their share by aggressively looking for methods that require relatively less labour. Entrepreneurs always encourage research and development in cost reducing methods of production, but when wages are expected to grow faster, they may particularly encourage invention of techniques that lower labour requirement per unit of output more than capital requirement. Clearly, bias from this source cannot be explained by changes in current relative prices or output. This type of bias is driven by expected relative price changes, and is manifested only over time.

Uselding and Juba (1973, Table A-2) provide numerical estimates of this kind of bias in the 19th century US manufacturing. They conclude: "*Over the long run technical progress was found to be 'labour saving', subject to the qualification that the underlying production for American manufacturing over this period* 1839-99 *was not Cobb-Douglas, that technical* *progress itself was non-neutral and that the elasticity of substitution was less than* 0.9". The qualifications could not be tested because of 'data limitations'. For all the required variables, a complete set of only seven decade end year observations were available for US manufacturing during the 19th century. This is an unavoidable limitation when having to work with historical data obtained as a result of ten-yearly censuses or even less frequent measures. For example, Broadberry and Gupta (2006) analyse wages, prices and economic development in Europe and Asia using observations at 50-year intervals during 1500-1800. Nonetheless, Uselding and Juba (1973, p.58) made an attempt to estimate the elasticity of substitution and test for non-neutrality, but the regressions – based on either the wage growth or the capital rental price growth equation – were too weak to yield reliable results.

The purpose of this paper is to illustrate the use of additional theoretical information and statistical techniques designed for estimation and tests in a small sample setting. These methods allow the Uselding and Juba (1973) qualifications to be tested using the same 'limited' data set. In addition, the suggested procedure, as set out in the following section, provides estimates of the extent of factor substitution and the bias of technical change.

2. A Model of Factor Substitution and Biased Technical Progress

We assume that U.S. manufacturing during the 19th century can be represented by an aggregate value added production function

$$V = F(K^*, L^*), \qquad (1)$$

where V = value added, K^* = capital services measured in efficiency units, L^* = labour services measured in efficiency units, and F is a linearly homogeneous function. It is well known that a value added specification is valid only if capital and labour are weakly separable from other materials. However, this hypothesis is not tested here because data on the price and the value of raw materials are not available for the particular decadal data set analysed. Cain and Patterson (1981) relax the value added specification with a four input (labour, capital, materials, others) annual data set for US manufacturing (1850-1919) at the 2-digit classification level. Their results show considerable variation in bias from industry to industry, but their finding of labour saving bias overall is consistent with the result based on the two-input model in this paper.

If producers minimize the cost of producing a specified amount of value added subject to given input prices, the value added function (1) can be completely represented by its dual cost function

$$C_V = f(p_K^*, p_L^*)V , \tag{2}$$

where C_V = cost of adding the value, p_K^* = price of augmented capital, p_L^* = price of augmented labour, and f is a linearly homogeneous and concave function.

We specify f to be the translog functional form of Christensen, Jorgenson and Lau (1971). When modified to incorporate input price diminishing technical change (e.g., see Woodward, 1983), the function is

$$\begin{aligned}\ln\left(\frac{C_V}{V}\right) &= \alpha_0 + \alpha_K \ln p_K^* + \alpha_L \ln p_L^* \\ &\quad + 0.5\beta_{KK}(\ln p_K^*)^2 + 0.5\beta_{LL}(\ln p_L^*)^2 + \beta_{KL} \ln p_K^* \ln p_L^* ,\end{aligned} \tag{3}$$

where

$$p_K^* = p_K e^{r_K t} \text{ and } p_L^* = p_L e^{r_L t} . \tag{4}$$

The coefficients r_K and r_L are the rates of diminution over time (t) of the natural input service prices p_K and p_L owing to technical progress. Defining g_K and g_L as the corresponding rates of factor augmentation, we have $K^* = Ke^{g_K t}$ and $L^* = Le^{g_L t}$. Since the value of input services must be invariant to units of measurement of factor augmentation, i.e., $p_K^* K^* = p_K K$ and $p_L^* L^* = p_L L$, we must have $g_K = -r_K$ and $g_L = -r_L$.

Linear homogeneity in prices imposes the following restrictions on (3),

$$\alpha_K + \alpha_L = 1, \ \ \beta_{KK} + \beta_{KL} = 0, \ \text{ and } \ \beta_{KL} + \beta_{LL} = 0. \qquad (5)$$

Substituting (4) and (5) into (3) we obtain

$$\ln(\frac{C_V}{p_K V}) = \alpha_0 + \alpha_L \ln\frac{p_L}{p_K} + 0.5\beta_{LL}(\ln\frac{p_L}{p_K})^2 + \alpha_T t + 0.5\beta_{TT}t^2 + \beta_{LT}(\ln\frac{p_L}{p_K})t, \qquad (6)$$

where

$$\alpha_T = r_K + \alpha_L(r_L - r_K), \ \ \beta_{LT} = \beta_{LL}(r_L - r_K), \text{ and } \ \beta_{TT} = \beta_{LL}(r_L - r_K)^2. \qquad (7)$$

The Cobb-Douglas specification is obtained if $\beta_{LL} = 0$. In this case, neither the bias nor the rates of factor augmentation are identifiable. Technical progress is said to be Hicks-neutral when $r_L = r_K$.

By applying Shepherd's (1953) Lemma in (6), we obtain the input share equations

$$s_L = \alpha_L + \beta_{LL}\ln\frac{p_L}{p_K} + \beta_{LT}t,$$

$$s_K = \alpha_K - \beta_{LL}\ln\frac{p_L}{p_K} - \beta_{LT}t,$$

where s_L and s_K are the cost shares of labour and capital implied by the optimal demands for these inputs. Another equation derived from (6) is

$$r = \alpha_T + \beta_{LT} \ln \frac{p_L}{p_K} + \beta_{TT} t ,$$

where $r = \frac{\partial \ln C_V}{\partial t}$, called the rate of cost diminution, is a measure of the rate of technical progress. It has a negative value for technical advancement. For a technology satisfying constant returns to scale, $\frac{\partial \ln C_V}{\partial t} = -\frac{\partial \ln V}{\partial t}$, where $\frac{\partial \ln V}{\partial t}$ is known as the rate of total factor productivity growth. It is a measure of technical change based on the underlying production function. This equivalence does not require neutrality of technical progress.

The price responsiveness of input i (= *L* or *K*) to the price of input j (= *L* or *K*) at a given level of output is measured by the Allen partial elasticity of substitution,

$$\sigma_{ij} = \frac{\beta_{ij} - s_i(\delta_{ij} - s_j)}{s_i s_j} , \tag{8}$$

where $\beta_{ij} = \beta_{ji}$ and $\delta_{ij} = 1$ if $i = j$ or 0, if not. For the Cobb-Douglas specification ($\beta_{KL} = 0$), the cross elasticity $\sigma_{KL} = 1$. It is < 1 if the parameter $\beta_{KL} < 0$.

The share equations show that bias of technical progress can be characterised as share saving or share using depending on the sign of $\frac{\partial s_i}{\partial t}, i = L, K$. For example, if $\beta_{LT} < 0$, technical progress is labour share saving (and hence necessarily capital share using in the two-input model). Bias of technical progress is usually defined in terms of the relative factor augmentation rates. The two definitions are of course equivalent when production is homothetic, e.g., when $\sigma_{KL} < 1$(i.e., $\beta_{KL} < 0$ in our model), technical progress is labour saving if $g_L > g_K$ (i.e., $r_L < r_K$). In this case, $\beta_{LT} = -\beta_{KL}(r_L - r_K) < 0$ (i.e., bias is labour share saving as well).

3. Data

Data required to estimate the coefficients in our model are the input prices p_K and p_L, the labour share s_L, and the rate of cost diminution r. Data on these variables, and price of value added p_V, are presented in Table 1. For the time variable t we use the numbers 1 to 7. Data on share of labour s_L are from Budd (1960, p. 382), who estimates the shares from 1850 to 1910. The shares for 1850-1900 are reported in Table 1 as at the decade end years 1849-1899. The share of labour rises from less than 55% at the end of the 1840s and 1850s to about 60% or more in the following decades. To reflect this trend, we estimate the share of labour in the 1830s, the decade immediately prior to the 1840s and 1850s, to be the mean value over these two decades. This value by itself is not used in our actual regression; the estimated share enters only as part of the share weight in calculating the non-parametric measure of growth of total factor productivity from 1839-1849.

The measure of price of value added p_V is from Gallman (1960, p. 43), obtained by deflating manufacturing value added in current prices by manufacturing value added in 1879 prices. Price of the capital input is calculated as $p_K = (1 - s_L) p_V V / K$, where V and K are quantities of value added and capital services. Data on these variables appear in Table 2. The p_K values calculated in this way are finally adjusted to have a value of 1 in 1879. Uselding and Juba 's (1973, p. 65, Table A-1) rental price index (expressed with a value of 1 in 1879) is: 0.832, 0.931, 1.000, 1.058, 1.000, 0.584 and 0.202. Their wage index with the corresponding values 0.699, 0.837, 0.874, 0.909, 1.000, 1.341 and 1.449 implies the wage-rental ratios 0.840, 0.899, 0.874, 0.859, 1.000, 2.296, and 7.173. The sudden jump in this series in the last decade is suspicious. One likely source of error is the unnecessary inflation of the 'price anticipation term' (Uselding and Juba, 1973, p. 67) by a factor of 5. An attempt to reconstruct their rental index failed because the underlying price index of capital goods in Brady (1967, Table 2b, p.110-111), which is the source they mention, does not contain the price index for consecutive years as required by their formula on p. 67. Fortunately, Uselding and Juba's (Table A-1, p.65) manufacturing capital series can be used to obtain the alternative rental price index that we use.

Price of labour p_L is obtained by deflating Lebergott's (1960, p. 462) male non-farm daily money wages by the estimates of average hours worked per day reported in Cain and Paterson (1981, p. 356). Hours worked per day for the six decades from 1850-1900 are 11.0, 11.0, 10.5, 10.3, 10.0, and 9.8. For the missing decade (1840), we have assumed a value of 11.0 as in the following two decades. Again, this observation is used only in calculating the non-parametric total factor productivity index, not in our final regression. The hourly wages obtained in the above manner were finally expressed as an index with a value of 1 in 1880.

Some interesting features of the data over the sample period (1839-1899) are: First, the share of labour increases (hence the share of capital decreases). Second, the prices of output and capital services fall while wages rise. Third, the fall in the price of capital services was rather sharp, indicating that the rising wage-rental ratio was driven more by this factor than by growing wages.

With profit maximising behaviour, or cost minimising behaviour with constant returns to scale, the rate of cost diminution r can be measured non-parametrically as the rate of growth of price of output minus the rate of growth of an aggregate input price. When technical progress is non-neutral, this rate equals the share weighted sum of the rates of factor augmentation. The latter is the Divisia index of prices of capital and labour services. When the underlying cost function is of the translog form, as we assume it to be in our model, the Divisia index is well approximated by the Tornqvist index. Hence, we compute:

$$r = (\ln p_{vt} - \ln p_{v,t-1}) - 0.5 \sum_{i=L,K} (s_{it} - s_{i,t-1})(\ln p_{it} - \ln p_{i,t-1}) .$$

Since this measure involves first differencing, one observation (1839) is lost at the beginning of the sample.

Technical progress is indicated by cost diminution, i.e., a negative value of r. Our measures show technical advancement in most of the decades; the only major exception is the 1860s, the civil war period during which technical change appears to have been regressive. The normal course of output growth was severely disrupted during this decade, resulting in a sharp decline in measured total factor productivity. To control for this upheaval, we use a dummy variable (D = 1 in 1869, 0 otherwise) in the regressions.

The equations that need to be estimated are the two share equations, and the cost diminution equation. Errors in these equations are assumed to be additive. Since one share equation is not independent, we arbitrarily drop the capital share equation from the system of estimating equations. It is known that maximum likelihood parameter estimates are invariant to the deleted equation. Our estimating system is then:

$$\begin{aligned} s_L &= \alpha_L + \beta_{LL} \ln \frac{p_L}{p_K} + \beta_{LT} t + u_1 , \\ r &= \alpha_T + \beta_{LT} \ln \frac{p_L}{p_K} + \beta_{TT} t + \beta_{DT} D + u_2 . \end{aligned} \tag{9}$$

The second equation in this system brings in additional information that helps alleviate the estimation problems plaguing the regressions attempted by Uselding and Juba (1973). With six observations on each dependent variable in (9), the available system observations are now 12 rather than 6.

We assume that the vector of disturbances (u_1, u_2) in (9) is multivariate normally and independently distributed with zero mean and a constant non-singular covariance matrix. The free parameters $\alpha_L, \alpha_T, \beta_{LL}, \beta_{LT}, \beta_{DT}$ are chosen such that the log of the likelihood function is maximised. It can be seen from (7) that β_{TT}, being equal to $\beta_{LT}^2 / \beta_{LL}$, is not a free parameter.

If the maximum is global, the associated parameter estimates are known to be consistent, efficient and asymptotically normally distributed. However, since our sample size is small, with six observations on each of our two dependent variables, we compute the standard errors of our parameter estimates also by utilizing the bootstrapping technique (Efron, 1979; Efron and Tibshirani, 1993). This procedure is explained in section 4.

Data on quantities of value added and input services, input-output ratios (where output is value added in constant dollars), and capital labour ratio are shown in Table 2. Value added in constant dollars V are from Gallman (1960, p. 43), and manufacturing capital in constant dollars K are from Uselding and Juba (1973, p. 65). We assume that quantity of capital services is proportional to the stock. Using Uselding and Juba's rental price index, an alternative measure of quantity of capital services can obtained as $K = (1 - s_L) p_V V / p_K$. These values are: 0.132, 0.218, 0.377, 0.624, 0.783, 2.100 and 9.748, which imply the capital labour ratios: 0.677, 0.631, 0.701, 0.789, 0.664, 0.912 and 3.496. The systematic jump in the last decade possibly reflects an error in Uselding and Juba's rental price index. The quantity of labour services is calculated as $L = s_L p_V V / p_L$.

Additional features of our data are revealed by Table 2. The quantities of value added and capital services grew at a faster rate than that of labour services, with quantity of capital services exhibiting the most rapid growth. These trends also imply that the capital output ratio increased while the labour output ratio (reciprocal of average productivity of labour) declined during this period.

The changes are highlighted by the capital-labour ratio which grows rapidly; this is associated with a persistent rise in the wage-rental ratio as can be seen from Table 1. There are two possible factors that can explain this relationship in our model – input substitution and biased technical progress. We turn now to disentangle these competing influences.

4. Estimation and analysis

Maximum likelihood coefficient estimates of (9) along with respective standard errors and *t*-ratios are listed in Table 3. The rate of capital price diminution $\hat{r}_K$, which is the negative of the rate of capital augmentation, does not differ from zero significantly at the 5% level. Since the difference $\hat{r}_L - \hat{r}_K$ is significantly less than zero, the observation $\hat{r}_K = 0$ implies that technical progress is biased, but it is only the labour input which is augmented. Thus, the estimates in Table 3 provide support for the hypothesis of labour saving technical progress in US manufacturing during the 19th century. Indeed, our results indicate that it was only the labour input which is augmented.

When we set $r_K = 0$, the log-likelihood value falls slightly from 20.177 to 20.150, which reflects the fact that the estimated coefficient $\hat{r}_K$ does not differ from zero significantly. The coefficients estimated under this restriction are shown in the first three columns of numbers in Table 4. These results indicate that the hypothesis of Cobb-Douglas technology $\beta_{LL} = 0$ can be rejected. The estimate $\hat{\beta}_{LL}$ is significantly different from zero at the 5% level.

Since our sample size is small, and the standard errors used for the test above are only asymptotic, we now turn to generating empirical distributions of the coefficient estimates, and of their *t*-ratios, that can be used for hypothesis tests by using the bootstrapping technique pioneered by Efron (1979, 1982). One of these methods, known as *case re-sampling* or *non-parametric bootstrapping*, draws random samples of the same size with replacement from the available sample of observations on the related variables. The model coefficients are then estimated repeatedly from these new samples where some of the observations may be repeated, while some may not occur at all. Besides a loss of some information, which may not matter when the sample size is not small and the number of replications is large, estimation may also fail if the sample size is small as in our case, and too many observations are repeated in any particular sample. For this reason, we opt for the other widely used method known as *residual re-sampling* (e.g., see Cameron and Trivedi, 2008, p. 360).

The steps involved in applying the latter method to our paper are:

(1) The parameter values are set at the estimated ones, i.e., Table 4 values in our case. The resulting pairs of residuals are assumed to be independently and identically distributed.

(2) From this distribution of the six paired residuals, a new random sample of paired errors of size six is drawn with replacement, i.e. some of these pairs of errors may be repeated or may not occur at each drawing.

(3) A new sample of the dependent variables – labour share and rates of cost diminution in our model – is generated using this sample of disturbances, the parameter values set in step 1, and the observations of the independent variables. A new set of regression coefficients is then obtained by using the new sample of the dependent variable, and the observed values of the covariates. It may be noted that, unlike the case re-sampling method, none of the observed values of the covariates is dropped in this regression.

(4) Steps 2 and 3 are then repeated sufficiently to construct empirical distributions of the estimated coefficients and other required statistics.

Results from 10,000 replications of the above procedure are listed in the bootstrap columns of Table 4. The standard deviations of the empirical distributions of the coefficients are the bootstrap estimates of the standard errors of the coefficients originally estimated. These are reported as Bootstrap Standard Error. Compared to these estimates, the asymptotic standard errors – listed as Standard Error in Table 4 – are all understated. A value of zero is still excluded by the 95% confidence intervals of the bootstrap distributions of all coefficients except β_{LL}. Thus, the hypothesis $\beta_{LL} = 0$ cannot be rejected in favour of the two-sided alternative $\beta_{LL} \neq 0$ at the 5% level of significance of the empirical distribution. However, the proportion of the bootstrap estimates of β_{LL} that are not positive is only about 0.053, suggesting an overwhelming likelihood that $\beta_{LL} > 0$ or $\beta_{LK} = -\beta_{LL} < 0$, i.e., that the production technology is not Cobb-Douglas.

This is one of the assumptions that Uselding and Juba (1993) could not support using their weak regression results. Our illustration shows that firmer statistical support is possible by using additional information offered by the equation for rate of cost diminution as part of the system in conjunction with the method of bootstrapping.

The original coefficient estimates in Table 4 are used to obtain the results reported in Table 5. The estimated cost function is well behaved. We impose linear homogeneity in prices and symmetry of substitution effects as our maintained hypotheses. The estimated function also satisfies the properties of monotonicity and concavity in prices at each observation. This is indicated by the estimated shares and the Allen partial elasticities of substitution. The estimated shares are positive, and the two inputs have negative own elasticities and are substitutes as required by concavity.

A number of important results appear in Table 5:

(i) Capital and Labour were not highly substitutable. The estimated elasticity of substitution, $\hat{\sigma}_{LK}$ using equation (8), are within the interval 0.3 to 0.4. At the mean labour share of 0.593, the elasticity is

$$\sigma_{LK} = 1 + \frac{\beta_{LK}}{s_L s_K} = 1 + \frac{-0.153}{0.593 \times (1 - 0.593)} = 0.366 .$$

(ii) The own elasticities of substitution, with average values of –0.252 and –0.532 for labour and capital respectively, indicate that capital was more responsive to its own price than labour. Thus, given the persistent fall in the rental price of capital, and in the rental wage ratio, during 1839-1899, the use of capital increased at a faster rate than that of labour owing to the substitution effect. As established earlier, this trend was reinforced by labour saving bias of technical progress.

(iii) The estimated rates of cost diminution indicate that total factor productivity grew by about 15% per decade, i.e., at about 1.4% per annum. The annual rate is computed as $g = \exp[\frac{1}{10}\ln(1+r)]-1$, where r is the rate of total factor productivity growth per decade. The estimate for the civil war decade (i.e., decade ending 1869) is simulated for $D=0$, i.e., it is the rate at which total factor productivity would have grown during that decade had the civil war not occurred. This value ($-r$ =15.6% per decade) implies that the apparently regressive technical change based on the non-parametric measure (in Table 1) was indeed an exogenous shock to output growth. This hypothesis is supported by the significance of the dummy variable coefficient β_{DT} in Table 3 (and 4).

Our results provide empirical support for Uselding and Juba's (1973) assumptions, for US manufacturing during the 19th century, that technical progress was non-neutral, that the production technology was not Cobb-Douglas, and that the elasticity of substitution between labpour and capital was less than 0.9. Additionally, our method is able to offer a point estimate of the elasticity of substitution between labour and capital (σ_{LK}), and of the rates of factor augmentation. The finding on bias of technical progress is consistent with the widely held belief that innovations during this period were labour saving, but our result goes a step further by establishing that technical advancement was most likely to have been labour augmenting only.

Assuming a CES technology with Hicks-neutral technical change, and cost minimising behaviour, Schmitz (1981) estimated σ_{LK} to range from 0.37 to 0.62 on average in his cross-section study of US manufacturing industries, at the 2-digit classification level, in 1860, 1870 and 1880. Our estimates in the range 0.3 to 0.4 for aggregate US manufacturing during 1849-1899, though not directly comparable to Schmitz's values, are based on the more general translog technology with non-neutral technical progress.

Since we do not impose the restriction of Hicks-neutrality, our model allows the factor ratio to change owing to both input substitution (σ_{LK}) and non-uniform technical progress. By imposing Hicks-neutrality we obtain larger values of 0.64 to 0.72 for σ_{LK}, similar to the larger estimates of Schmitz (1981), but these estimates are probably biased. This restriction, not acceptable in our model, forces the capital-labour ratio to change owing to the substitution effect only, resulting in an upward bias in the elasticity estimate.

Kendrick's (1961, Table D-1, p. 464) investigation of sector-wise factor productivity growth gave an average rate of 14.1% per decade, i.e., a compound rate of 1.3% annually, for the manufacturing sector during 1869-1899. According to our estimates for the period (1849-1899), which adjust for the disruption to growth caused by the civil war, total factor productivity grew at about 1.4% per annum. A measure not adjusted for exogenous shocks would yield a smaller rate of growth. For example, the average of our non-parametric measure including the civil war decade is 6.2% per decade, i.e., about 0.6% per year. With the civil war decade excluded, the non-parametric measure is about 1.3% per annum which underlies our parametric estimate of 1.4% per annum.

5. Conclusion

Our illustration in this paper demonstrates how to alleviate information requirement on small datasets

- by incorporating additional theoretical information, if any are available, and
- by using the method of bootstrapping.

This approach can be applied generally to many similar situations. In the present illustration, the method is able to offer econometric support to the untested assumptions made in Uselding and Juba (1973).

Additionally, our approach offers more precise numerical measures of the coefficients in the range or direction assumed. For example, we find

- that, for a value added technology with non-neutral technical progress, the elasticity of substitution between labour and capital was in the range of 0.3 to 0.4 in aggregate US manufacturing during the 19th century, and
- that technical progress during the nineteenth century was labour saving. Additionally, we find that the change was labour augmenting only.

References

Broadberry, S. and B. Gupta (2006), "The early modern great divergence: wages, prices and economic development in Europe and Asia, 1500-1800", *Economic History Review*, Vol. 59, pp. 2-31.

Budd, E.C. (1960), "Factor Shares, 1850-1900" in *Trends in the American Economy in the Nineteenth Century*, NBER, Vol. 24.

Cain, L.P. and D. G. Patterson (1981), "Factor Biases and Technical Change in Manufacturing: The American System, 1850-1919", *Journal of Economic History*, June, Vol. 41, No.2.

Cameron A. C. and P. K. Trivedi (2005), *Microeconometrics: Methods and Applications*, Cambridge: Cambridge University Press.

Christensen, L., D. Jorgenson and L. J. Lau (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function," (abstract), *Econometrica*, July, Vol. 39, pp. 255-256.

Efron, B. (1979), "Another Look at the Jack-knife", *Annals of Statistics*, vol.7, p.1-26.

Efron, B. and R. Tibshirani (1993), *An Introduction to the Bootstrap*, New York: Chapman & Hall.

Gallman, R.E. (1960), "Commodity Output, 1839-1899" in *Trends in the American Economy in the Nineteenth Century*, NBER, Vol. 24.

Kendrick, J. W. (1961), *Productivity Trends in the United States*, Princeton: Princeton University Press for the National Bureau of Economic Research.

Rothbarth, E. (1946), "Causes of the Superior Efficiency of U.S.A. Industry as Compared with British Industry," *Economic Journal*, September, Vol. 56, pp. 383-390.

Shepherd, R.W. (1953), *Cost and Production Functions*, Princeton: Princeton University Press.

Schmitz, M. (1981), "The Elasticity of Substitution in 19th-Century Manufacturing", *Explorations in Economic History*, Vol.18, p.290-303.

Uselding, P. and B. Juba (1973), "Biased Technical Progress in American Manufacturing, 1839-1899," *Explorations in Economic History*, Fall, Vol. 11, pp. 55-72.

Woodward, T. (1983), “A Factor Augmenting Approach for Studying Capital Measurement, Obsolescence, and the Recent Productivity Slowdown”, in *Developments in Econometric Analysis of Productivity*, A. Dogramaci, editor, Boston: Kluwer-Nijhoff Publishing.

Table 1: Share of Labour, Prices and Rate of Cost Diminution per Decade
in US Manufacturing, 1839-1899

Decade End Year	s_L	p_V	p_K	p_L	r	p_L / p_K
1839	0.541*	1.263	1.680	0.622	...	0.370
1849	0.545	0.916	1.749	0.658	−0.371	0.377
1859	0.537	0.949	1.700	0.761	−0.030	0.447
1869	0.595	1.513	1.243	1.203	0.343	0.968
1879	0.601	1.000	1.000	1.000	−0.216	1.000
1889	0.671	0.897	0.597	1.119	0.007	1.873
1899	0.609	0.805	0.559	1.158	−0.106	2.070

* This missing observation is estimated as the average of the next two observations.

Table 2: Quantities of Value Added and Inputs (in billions of 1879 dollars)
in US Manufacturing, 1839-1899

Decade End Year	V	K	L	K/V	L/V	K/L
1839	0.190	0.257	0.209	1.353	1.099	1.231
1849	0.488	0.456	0.370	0.934	0.758	1.232
1859	0.859	0.870	0.575	1.013	0.670	1.512
1869	1.078	2.084	0.807	1.933	0.748	2.584
1879	1.962	3.069	1.179	1.564	0.601	2.603
1889	4.156	8.050	2.236	1.937	0.538	3.600
1899	6.262	13.827	2.653	2.208	0.424	5.211

Table 3: Estimates of a Two Input Translog Technology
in US Manufacturing, 1839-1899

Coefficient	Estimate	Standard Error	*t*-ratio	*p*-value
$\hat{\alpha}_L$	0.736	0.061	12.11	<0.001
$\hat{\beta}_{LL}$	0.151	0.043	3.50	0.010
$\hat{r}_L - \hat{r}_K$	–0.248	0.042	–5.93	0.001
$\hat{r}_K$	–0.015	0.063	–0.23	0.821
$\hat{\beta}_{DT}$	0.600	0.140	4.30	0.004

Table 4: Estimates of a Two Input Translog Technology
with Labour-augmenting Technical Progress
in US Manufacturing 1839-1899

Coefficient	Estimate	SE*	*t*-ratio	*p*-value	Bootstrap SE	Bootstrap 95% interval
$\hat{\alpha}_L$	0.740	0.059	12.57	<0.001	0.111	(0.467, 0.946)
$\hat{\beta}_{LL}$	0.153	0.043	3.59	0.007	0.082	(–0.059, 0.296)
$\hat{r}_L$	–0.251	0.039	–6.36	<0.001	0.112	(–0.538, –0.036)
$\hat{\beta}_{DT}$	0.591	0.135	4.38	0.002	0.246	(0.184, 1.040)

Table 5: Estimated Labour Shares, Allen Partial Elasticities, and
Rates of Cost Diminution in U.S. Manufacturing, 1849-1899

Decade End Year	$\hat{s}_L$	$\hat{\sigma}_{LL}$	$\hat{\sigma}_{KK}$	$\hat{\sigma}_{LK}$	$\hat{r}$
1849	0.552	–0.310	–0.470	0.382	–0.139
1859	0.540	–0.327	–0.451	0.384	–0.136
1869	0.620	–0.216	–0.572	0.351	–0.156
1879	0.586	–0.261	–0.524	0.370	–0.147
1889	0.644	–0.184	–0.602	0.333	–0.162
1899	0.621	–0.214	–0.574	0.351	–0.156

TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON