

Bubbles rising in line: champagne, lager, cider

J. F. Harper

School of Mathematics, Statistics and Computer Science,
Victoria University of Wellington
PO Box 600, Wellington 6140, New Zealand

Abstract

Various drinks contain vertical lines of rising bubbles. They are usually stable in glasses of champagne, lager or cider unless two lines are near enough to interfere with each other.

It is already known that in a pure liquid the lines are unstable, because the wake of a rising bubble contains rising liquid, so the vortex rings in it are horizontal. A second bubble rising off the centreline will make those rings slope up on one side of its own wake and down on the other, revealing the existence of a lift force on that bubble pushing it further out of line, in the same way that the lift force supporting an aircraft is revealed by the vortices trailing behind its wings. It is also known that in a surfactant solution the lines are stabilised, because the first bubble has a higher concentration of surfactant on the centreline of its wake than further out, which makes the surface tension lower there, and it is therefore energetically favourable for the second bubble to follow the first. Hence the second bubble may move out of line in a sufficiently dilute surfactant solution, but remain in line in a more concentrated solution. This conclusion is unaltered if a bubble follows behind several previous ones, though the critical concentration for neutral stability will change.

That theory was for bubbles whose size remains the same as they rise. Real bubbly drinks are usually supersaturated solutions of carbon dioxide, which makes the bubbles grow as they rise. Each growing bubble then acts like a point source of liquid, and any two of them repel each other. If a lower bubble is slightly out of line with those above it, it will tend to move further out of line. That effect increases the amount of surfactant needed to stabilise the line, but not enough to destabilise it in ordinary drinks.

1. Introduction

Before considering a line of bubbles like those in Figure 1, let us begin with a single bubble of radius a , rising at speed U in a liquid of kinematic viscosity ν . Suppose that the bubble is

- (1) large enough that its Reynolds number $Re \gg 1$, where $Re = 2Ua/\nu$, but also
- (2) small enough to be very nearly spherical, and that
- (3) the liquid is so pure that surface-active solutes do not appreciably slow down the bubbles.

All three conditions can be, but often are not, satisfied. For example, Bryn (1933) did experiments that obeyed them for bubbles with $0.5 \text{ mm} < a < 1 \text{ mm}$ in a 13% solution of ethyl alcohol in water, which is within the strength range for champagne, but his water without alcohol was not clean enough to satisfy condition 3. His water must have contained some adventitious surfactants, but his alcohol presumably made the surfactant less surface-active.

The three conditions imply (Moore 1963) that the flow around a bubble is irrotational except for thin viscous boundary layers around its surface and in its wake, and if ρ is the liquid density, the drag coefficient C_D is

$$C_D = (\text{drag force}) / (0.5\pi a^2 \rho U^2) \approx 48(1 - 2.21Re^{-1/2}) / Re \quad (1)$$

Theories exist for three physical mechanisms relevant to lateral stability of bubble wakes: vortex-induced instability (Harper 1970), surfactant-induced stability, for which an order-of-magnitude estimate can be calculated from Lerner & Harper (1991), and growth-induced instability, which can be estimated from Harper (2001). The purpose of this paper is to bring together for the first time the results of all three mechanisms. Experimental data are available for champagne (Liger-Belair *et al.* 2000), and the theory is tested on them. Much more information about champagne, at a less technical level, is in his book (Liger-Belair 2004).

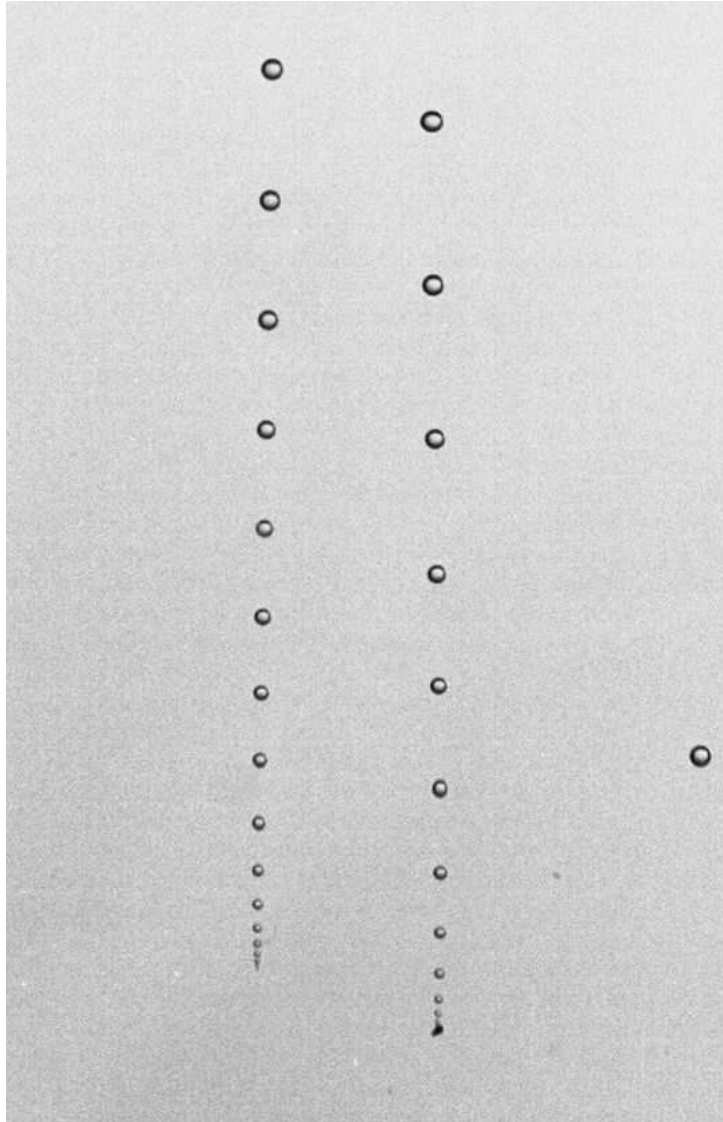


Figure 1. Bubbles rising in a cylindrical champagne flute. (Courtesy Gérard Liger-Belair.)

2. Instability revealed by wake vortices

Harper (1970) considered a second bubble rising under the first, but shifted a small distance sideways, and obeying the three conditions. Let the vertical distance between the bubble centres be ℓ , where $a \ll \ell \ll aRe^{1/2}$, and let the second bubble move to the left a distance d . If $d \ll aRe^{-1/4}$ the first bubble's wake, which consists of vortex rings as shown in Figure 2, washes over the whole of the second. The rings pass nearer to its top stagnation point on its left side, so they take longer to go around it there. The rings are then no longer horizontal, but go up one side of the wake and down the other. That reveals the existence of a lift force, in the same way that an aircraft's trailing vortices do. The wings push air down, and that generates vortices rotating as shown, with a vortex line from the right wing pointing away from the aircraft, and one from the left wing pointing towards it.

The lift coefficient $C_{L\omega}$ revealed by these trailing vortices turns out to be

$$C_{L\omega} = (\text{sideways force}) / (0.5\pi a^2 \rho U^2) \approx 14.4 Re^{-1/2} d/a \quad (2)$$

The force is in the direction pushing the bubble further out of line, so lines of rising bubbles should be unstable. There must be another effect causing the stability observed in many drinks, e.g. champagne and lager, and the most likely candidate appears to be surface activity.

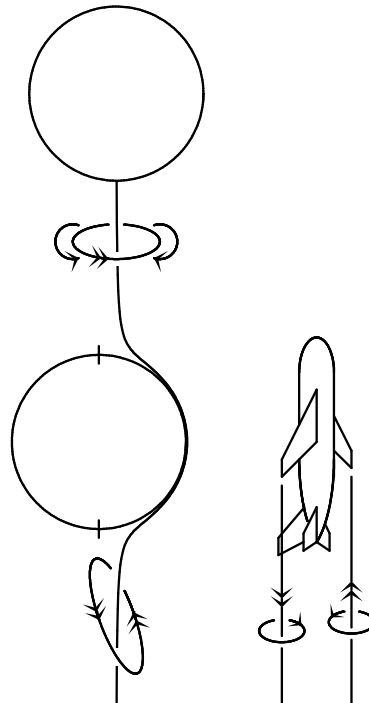


Figure 2. The vortex system for a pair of bubbles in a pure liquid, showing the analogy between the trailing vortices and an aircraft's; in both cases the downward-pointing vorticity vector is nearer to the viewer than the upward-pointing one, and the lift force is to the left. Single arrows: flow induced by vortices. Double-headed arrows: the vorticity vector.

3. Surfactant wake stabilisation

The surface pressure Π is defined as $\sigma_p - \sigma$, where σ_p, σ are the surface tensions of the pure liquid and the actual liquid; Π and σ vary around a bubble. In an ideal solution,

$$\Pi = RT hc \quad (3)$$

where R is the gas constant, T is the absolute temperature, h is the adsorption depth, which is a constant with the dimensions of length that characterises the surfactant, and c is the local surfactant concentration in the liquid. Even where there is no free surface, Π may be defined by (3). Far from the bubbles let $\Pi = \Pi_\infty$. Also, let ψ be the stream function of the motion past the first bubble, ignoring the second (and any subsequent) bubbles. Then Lerner & Harper (1991) showed that Π in the wake of the first bubble varies with ψ as shown in Figure 3.

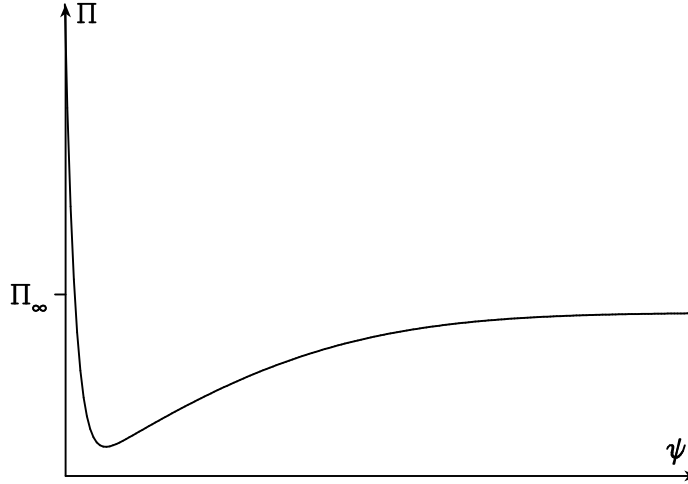


Figure 3. Schematic variation of surface pressure Π across the first bubble's wake, plotted against stream function ψ .

If Pe is the Péclet number $2Ua/D$, where D is the surfactant diffusivity in the liquid, the height of the narrow “spike” on the centreline is of order $\Pi_\infty Pe^{-1/6}$, and its width is of order $Ua^2 Pe^{-2/3}$ as a function of $\psi \approx \frac{1}{2}Ur^2$, where r is the cylindrical polar distance from the wake centreline. Hence the spike has a physical width of order $aPe^{-1/3}$.

If the second bubble is a distance $d \ll aPe^{-1/3}$ to the left of the first, then Π on its right side is $O(\{\Pi_\infty Pe^{1/6} d\} / \{aPe^{-1/3}\}) = O(\Pi_\infty Pe^{1/2} d/a)$ higher than on its left side. There is therefore a horizontal shear stress around the second bubble, drawing the fluid on its surface towards regions of higher surface tension, or lower Π . The reaction on the bubble is a lift force F_L of order $\Pi_\infty Pe^{1/2} d$ propelling the bubble to the right, i.e. back into line with the bubble above it. The lift coefficient $C_{L\Pi}$ due to surface activity is then

$$C_{L\Pi} = O\left(\{\Pi_\infty Pe^{1/2} d\} / \{a^2 \rho U^2\}\right) \quad (4)$$

4. Bubble repulsion due to growth

In bubbly drinks the bubbles grow as they rise because they are in a liquid supersaturated with a gas (usually carbon dioxide, but sometimes nitrogen is added.) If there are two bubbles in a vertical line, each of radius a increasing at a rate \dot{a} , and the fluid motion is irrotational, and the first and second bubbles' centres are at heights q_1, q_2 above some fixed level, so that $\ell = q_1 - q_2$, then (Harper 2001) Lagrange's equations reduce to

$$\ddot{q}_1 + 3v(6 + \alpha)\dot{q}_1/a^2 = 2g + 9v\dot{a}(2 + \alpha)/\ell^2 \quad (5)$$

$$\ddot{q}_2 + 3v(6 + \alpha)\dot{q}_2/a^2 = 2g - 9v\dot{a}(2 + \alpha)/\ell^2, \quad (6)$$

if terms of order $(a/\ell)^3$ and higher are ignored; $\alpha = a\dot{a}/v$, a Reynolds number for bubble growth.

In champagne a is of order 0.5 mm or less, U is of order 30 mm/s, \dot{a} is of order 0.25 mm/s, and $v = 1.66 \text{ mm}^2/\text{s}$, (Liger-Belair *et al.* 2000), so that $\alpha < 0.1$. For a first approximation we may ignore the terms involving α in (5) and (6).

Equations (5) and (6) involve accelerations; the corresponding forces may be found by multiplying by the bubble's virtual mass $2\pi\rho a^2/3$. The force of repulsion that each bubble exerts on the other due to its growth is thus $12\pi\rho v\dot{a}a^3/\ell^2$. The horizontal component of that force is $F_G = 12\pi\rho v\dot{a}a^3 d/\ell^3$, so the lift coefficient C_{LG} due to growth is

$$C_{LG} = F_G/(\frac{1}{2}\pi\rho a^2 U^2) \approx 6.7\alpha Re^{-3/2}(a/\ell)^3 C_{L\omega}. \quad (7)$$

Because α , $Re^{-3/2}$ and $(a/\ell)^3$ are all much smaller than 1, it appears that the sideways force due to growth may be safely ignored, at least in champagne.

5. Results and discussion

The significance of the calculations presented here is most easily appreciated by finding how much surfactant is needed to stabilise a line of bubbles. By (7) we may ignore C_{LG} , and then (2) and (4) show that $C_{L\Pi} > C_{L\omega}$, and the line is stable, if

$$\Pi_\infty > \Pi_{\text{crit}} = k\rho U v^{1/2} D^{1/2}, \quad (8)$$

where k is a constant of order 1; its exact value has not been obtained. In the champagne of Liger-Belair *et al.* (2000) the surfactant was a protein with $D = 6.7 \times 10^{-11} \text{ m}^2/\text{s}$, so that $\Pi_{\text{crit}} = 0.32k \mu\text{N}/\text{m}$, and the surface concentration was about $\Gamma = 0.3 \text{ mg}/\text{m}^2$. If the surfactant had a mass of 67 kg/mol as reported for bovine serum albumin (Kroschwitz 1998), then $\Pi_\infty = 33 \mu\text{N}/\text{m}$ (Ybert & di Meglio 2000, Henderson 1975). The stable lines of bubbles that were observed suggest, then, that $k < 100$. Note that as $\sigma_p = 47 \text{ mN}/\text{m}$, Π_∞ is a change of surface tension by about 1 part in 1400. As is well known, bubbles provide an extremely sensitive test of surface contamination.

Liger-Belair *et al.* (2000) also gave some results for beer, which has much higher surfactant concentrations than champagne. As one would expect, bubbles in beer of a given size rose more slowly than in champagne. On the present theory, lines of bubbles in beer are even more stable than in champagne unless they are close enough to interfere with one another. Other bubbly beverages, such as cider, still await determination of their surfactant concentrations.

6. Conclusions

Distortion of originally horizontal vortex rings by a bubble rising not exactly beneath a previous bubble is a mechanism for instability, but surfactants in the previous bubble's wake can keep the next bubble in line if present in high enough concentration. Even champagne, which has very little dissolved surfactant, has enough. Beer has much more, and stable vertical lines are indeed observed in beers such as lager in which there are few lines of bubbles. The theories of Sections 2, 3 and 4 assume conditions not exactly satisfied by real bubbles, but they do appear to be a useful way to understand a frequently-observed phenomenon.

Acknowledgements

I am most grateful to Gérard Liger-Belair for sending me a copy of Figure 1 and allowing me to reproduce it.

Nomenclature

$\dot{}$ (dot on variable)	Time rate of change of that variable
a	Bubble radius
c	Surfactant concentration in solution
C_D	Bubble drag coefficient defined in (1)
$C_{L\omega}, C_{L\Pi}, C_{LG}$	Bubble lift coefficients due to vorticity, surfactant, growth
d	Sideways displacement of second bubble
D	Surfactant diffusivity in liquid
F_G	Horizontal force component due to growth
h	Adsorption depth, Γ/c in ideal solution
k	Dimensionless constant in (8)
ℓ	Vertical distance between bubbles
Pe	Péclet number $2Ua/D$
q_1, q_2	Heights of bubble centres above a fixed level
r	Distance from wake centreline
R	Gas constant
Re	Reynolds number $2Ua/\nu$
T	Absolute temperature
U	Speed of rise
α	Bubble growth Reynolds number $a\dot{a}/\nu$
Γ	Surface concentration of surfactant, Π/RT in ideal solution
ν	Kinematic viscosity of liquid
π	3.14159...
Π	Surface pressure $\sigma_p - \sigma$
Π_∞	Equilibrium surface pressure
Π_{crit}	Critical value of Π_∞ for stability of line of bubbles
ρ	Liquid density
σ	Surface tension of actual liquid

σ_p	Surface tension without surfactant
ψ	Stream function of liquid flow past a single bubble

References

- Bryn T (1933). Steiggeschwindigkeit von Luftblasen in Flüssigkeiten. *Forsch. Geb. Ingenieurw.*, 4:27–30.
- Harper J F (1970). On bubbles rising in line at large Reynolds numbers. *J. Fluid Mech.*, 41:751–758.
- Harper J F (2001). Growing bubbles rising in line. *J. Appl. Math. Decision Sciences*, 5:65–73.
- Henderson D (1975). A simple equation of state for hard discs. *Mol. Phys.*, 30:971–972.
- Kroschwitz J I, editor (1998). Kirk-Othmer Encyclopedia of Chemical Technology. Wiley, New York, USA.
- Lerner L, Harper J F (1991). Stokes flow past a pair of stagnant-cap bubbles. *J. Fluid Mech.*, 232:167–190.
- Liger-Belair G (2004). *Uncorked: The Science of Champagne*. Princeton University Press, Princeton, USA.
- Liger-Belair G, *et al.* (2000). On the velocity of expanding spherical gas bubbles rising in line in supersaturated hydroalcoholic solutions: Application to bubble trains in carbonated beverages. *Langmuir*, 16:1889–1895.
- Moore D W (1963). The boundary layer on a spherical gas bubble. *J. Fluid Mech.*, 16:161–176.
- Ybert C, di Meglio J M (2000). Ascending air bubbles in solutions of surface-active molecules: influence of desorption kinetics. *Eur. Phys. J. E*, 3:143–148.