Young Einstein and the beginnings of quantum mechanics
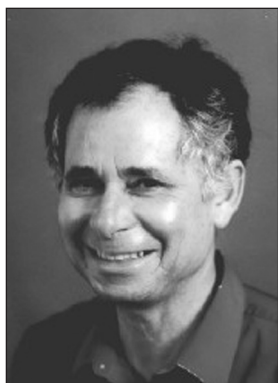
John Lekner

School of Chemical and Physical Sciences, Victoria University of Wellington, PO Box 600, Wellington

In 1905 Einstein wrote five history-making papers (on the particle nature of light, Brownian motion, special relativity, energy and inertia, and on molecular dimensions). He was 26, married to Mileva Marić, with one-year-old child Hans Albert, and employed as Technical Expert (third class) by the Bern Patent Office. His relativity theory captured the public imagination, but his quantum ideas were even more revolutionary. His light quantum (now the 'photon') was rejected by Planck, who is often thought of as the originator of the concept, and by Millikan, whose 1916 photoelectric data agreed beautifully with Einstein's predictions ('a bold, not to say reckless, hypothesis of an electromagnetic light corpuscle … which flies in the face of thoroughly established facts of interference'). Compton's 1923 paper on the scattering of X-rays and gamma rays uses the light quantum idea, but makes no mention of Einstein. There followed more than 20 papers on various aspects of quantum theory, substantial enough to make Einstein one of the founders of quantum mechanics, possibly the founder. This paper traces the beginnings of quantum mechanics, giving an outline of Einstein's contributions. It begins with a sketch of his early life.

Young Einstein (1879 to 1904)

Albert Einstein was born in Ulm, Bavaria, on 14 March 1879. His parents were Hermann and Pauline (nee Koch), 'of Israelite religion' as stated on Albert's birth certificate, actually non-practising Jews. Hermann was a partner in his cousin's mercantile business in Ulm. Hermann's younger brother Jakob was an engineer, and persuaded Hermann to join him in a plumbing and electrical venture in Munich, when Albert was two. Jakob had designed a dynamo, which he wanted to manufacture. The venture failed in Germany, but showed promise in Italy. The plant was accordingly transferred to Pavia, the family moving to Milan in 1894, later to Pavia, and then back to Milan. Eventually this failed, and the family had little left. Most of the (initially large) Koch family assets had been used up, but against Albert's wise advice (he was then 17), Hermann started up a third electrical factory, in Milan. This failed also. Hermann died of a heart condition in 1902, when Albert was 23. By then the family had experienced much mobility, and a sharp decline in wealth.

Albert's younger sister Maja (Maria) has written a charming and perceptive biographical sketch of Albert's early years [1]. As a child he was quiet and would play by himself for hours. He had 'such difficulty with language that those around him feared he would never learn to speak'. At 2½ he was told of the arrival of a little sister, with whom he could play. He must have imagined a kind of toy, because at the sight of the baby he asked, with disappointment, 'Yes, but where are its wheels?'

As a child of four or five his father brought him a magnetic compass (he was ill in bed). In his autobiography Einstein recalled the sense of wonder at how the enclosed and apparently isolated needle responded to an invisible magnetic field.
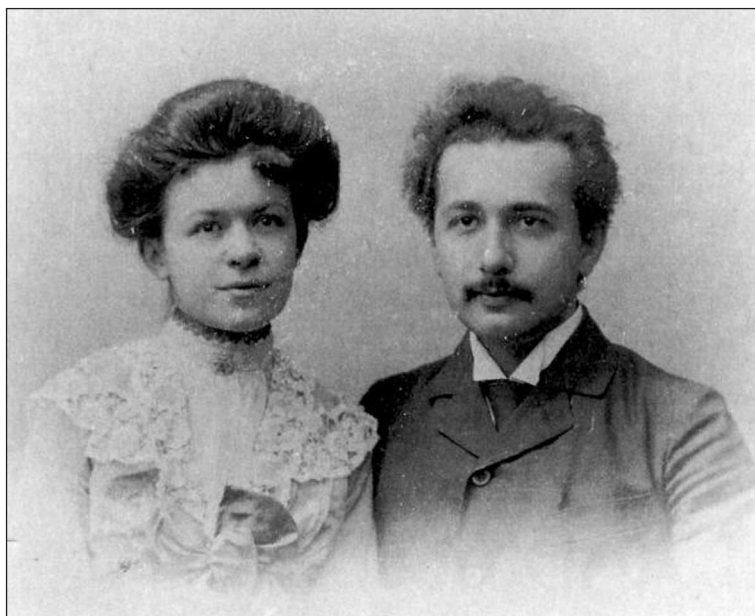
Though quiet, young Albert had a temper. A lady violin teacher was attacked with a chair, never to reappear. Maja herself was subject to his violent tantrums, once having a large bowling ball thrown at her; another time she was hit in the head with a child's hoe. She remarks ruefully '... a sound skull is needed to be the sister of a thinker'. The temper tantrums disappeared during his early school years (he entered school at 7).

Persistence and tenacity were already part of his character at this age. He would work on puzzles, erect complicated structures with his building block set, and build houses of cards up to 14 stories high.

On entering public school, religious instruction was obligatory. (He and Maja went to a nearby Catholic elementary school.) He was taught at home by a distant relative, and 'caught religion'. He ate no pork for years, and took it amiss that his parents were lax in their Jewish observances.

Young Albert was not happy at secondary school, the Luitpold Gymnasium (like a grammar school, where pupils receive a classical education). He later wrote 'As a pupil I was neither particularly good nor bad. My principal weakness was a poor memory ... especially for words and texts'. His teacher of Greek told him 'You will never amount to anything'. However, he was captivated by mathematics: earlier, his uncle Jakob (the engi-

John Lekner was educated at Auckland Grammar, University of Auckland, and University of Chicago. He has taught at Cambridge and at Victoria University of Wellington. He has written one book, 'Theory of reflection of electromagnetic and particle waves' (1987), and more than 100 papers, mainly in the fields of quantum mechanics, statistical physics, and electromagnetism. John may be contacted at john.lekner@vuw.ac.nz

Mileva Marić and Albert Einstein, wedding photograph, 1903.

neer) told him of the Pythagorean theorem; after considerable effort Albert found a proof. He later worked through a book on Euclidean geometry, the theorems in which had a 'lucidity and certainty (that) made an indescribable impression on me'. The exposure to mathematics and science now made the impressionable Einstein antireligious. The certainty of belief was to be replaced by a profound suspicion of any authority. This must have been apparent in class: his teacher of Greek asked him to leave the school: 'Your mere presence spoils the respect of the class for me'. At this time his parents had moved to Italy, Albert staying behind in Munich to finish his secondary education. He obtained a doctor's certificate to enable him to leave the Luitpold Gymnasium (at 15), and rejoined his family in Milan. One of the most joyous periods in his life followed, as he studied only the subjects he liked. 'Museums, art treasures, churches, concerts, books and more books, family, friends, the warm Italian sun, the free, warmhearted people – all merged into a heady adventure of escape and wonderful self-discovery' [3].

Albert resolved to change his citizenship: 'The over-emphasized military mentality in the German State was alien to me even as a boy'. He applied for Swiss citizenship when he was of age to do so (in October 1899); this was granted in February 1901. He completed his secondary schooling in Switzerland, and was admitted to the Zurich Polytechnic (ETH) in October 1896. (There he met fellow physics student Mileva Marić, from Serbia, whom he was later to marry. A daughter, called 'Lieserl' in their letters, was born out of wedlock early in 1902. It seems she was adopted; nothing further is known.) He graduated with a *Diplom* in July 1900, but was unable to obtain any work except tutoring and substitute teaching jobs for two years. Finally he secured a position as Technical Expert (third class) at the Swiss Patent Office in Bern in June 1902. His father died in October of that year, and he married Mileva in January 1903. Their son Hans Albert was born in May 1904.

Einstein's early publications (1901–1904) were on thermodynamics. The first quantum paper, 'On a heuristic point of view concerning the production and transformation of light' [4], was submitted in March 1905. This, and related papers, will be discussed in Section 3.

First clues
Spectral lines
We live in a quantum universe, but its quantum nature is not easily apparent. This section traces the first clues; all three originated in nineteenth century observations and experiments, and two were associated with light.

William Hyde Wollaston (1766–1828) noticed dark lines in the solar spectrum in 1802; he interpreted these as gaps separating the colours of the Sun. Joseph Fraunhofer (1787–1826) rediscovered these lines (now known to be absorption lines) in the course of precision measurements of the refractive index

Albert Einstein at Bern Patent Office.

of glass at various wavelengths. He made a catalogue of these 'Fraunhofer lines', to be used as calibration wavelengths. He also noticed that some of these lines corresponded to certain emission lines in sparks and flames.

However, it was not till about 1860 that the significance of the spectral lines as signatures of elements (atomic or ionised) was recognised, through the work of Robert Wilhelm Bunsen (1811–1899) and Gustav Robert Kirchhoff (1824–1887). Bunsen used the colour that salts gave to the flame of his burner as an analytical tool. His friend Kirchhoff suggested looking at the flames through a spectroscope. It was soon clear that Fraunhofer's lines were characteristic of chemical elements, and that the dark (absorption) lines in the solar spectrum were at the same wavelength as the bright (emission) lines of certain elements. Thus spectral analysis was born. One could deduce from the solar spectrum the composition of the Sun's surface layers! (This discovery came rather soon after the French philosopher August Compte (1798–1857), founder of positivism and social philosophy, wrote in 1844 that 'The stars are only accessible by distant visual exploration. This inevitable restriction not only prevents us from speculating about life on all these great bodies, but also forbids the superior inorganic speculations relative to their chemical or even their physical natures.') New elements were discovered by the method of spectral analysis: caesium ('blueish' in Latin), rubidium (red), and others in flame spectra, helium (from the Greek *helios*, the Sun) in the solar spectrum, in 1868, long before the element was discovered on Earth.

Why should there be sharp spectral lines, different for each element? Classical mechanics and electrodynamics dealt in continua. Yes, there can be resonances, but the spectral lines were sharper than any resolution available then, and classical resonances are typically broad. Regularities in the lines were noticed: Balmer (1825–1898) fitted the four visible lines in the hydrogen atom spectrum, at 410, 434, 486 and 656 nm, to

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad n = 3, 4, 5, 6$$
(1)

The constant R (now known as the Rydberg constant) is $R = 1.097 \times 10^7$ m^{-1}. Likewise the Lyman series $\lambda^{-1} = R(1^{-2} - n^{-2})$, $n = 2, 3, ...$ which lies in the ultraviolet, and the Paschen series $\lambda^{-1} = R(3^{-2} - n^{-2})$, $n = 4, 5, ...$ in the infrared, were seemingly perfect fits to the spectral wavelengths of hydrogen. Rather neat, but totally mysterious, and not even partially understood till 1913, when Niels Bohr (1885–1962), building on the quantum concepts of Max Planck (1858–1947) and Einstein, and using the atomic picture of our own Rutherford (1871–1937), finally produced a quantum model for the hydrogen atom.

Jumping ahead to 1913 [5], the assumptions of (i) quantisation of the angular momentum of the electron orbit in integer multiples of $\hbar = h/2\pi$ (h is Planck's constant, to be discussed below), and (ii) that the frequency f of the emitted radiation is given by Einstein's $\qquad hf = \Delta E \qquad$ (2)

where ΔE is the difference in energy between two allowed energy levels of the atom, enabled Bohr to derive a formula for all possible transitions of the hydrogen atom:

Niels Bohr (1885–1962), Nobel 1922.
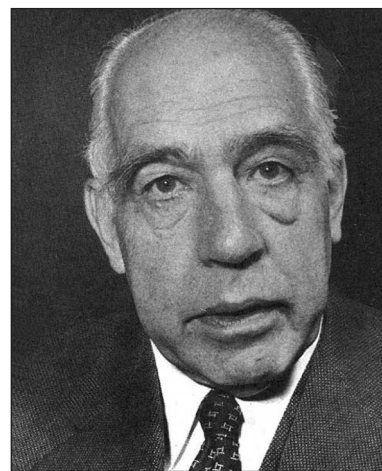
Ernest Rutherford (1871–1937), Nobel 1908.

$$\frac{f}{c} = \frac{1}{\lambda} = \frac{me^4}{4\pi c\hbar^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
(3)

Here m and e are the mass and charge of the electron, and c is the speed of light. This formula not only includes as special cases the Balmer, Lyman, Paschen, etc. series, but also gives a theoretical value of the Rydberg constant in terms of the fundamental constants c, $\hbar$, e and m.

Thermal radiation

When material bodies are heated they glow. The radiation emitted changes colour as the temperature is increased, from a dull red to orange to yellow. The light from our Sun ('white' light) has a mixture of wavelengths, from the infrared through the visible to the ultraviolet. The energy content is largest at around 500 nm, in the green part of the spectrum. We now know that the temperature of the Sun's surface is about 6000 K. A bar heater can be at 1000 K to 1500 K, a tungsten filament in an incandescent lamp radiates at about 2500 K. By 1859, James Clerk Maxwell (1831–1879) had discovered the law of distribution of velocities (and thus also of energies) in a gas in thermal equilibrium. Is there a corresponding law that gives the distribution of wavelengths (or frequencies) in thermal radiation?

There is, if we idealise to an absorber/radiator that absorbs all radiation incident on it (a 'black body'). A good approximation to a black body is a small hole in an enclosure, the walls of which are all at the same absolute temperature T. All radiation into that small hole will be absorbed, because it bounces many

Cosmic background radiation, fitted to Planck's formula.

times around the interior, with vanishing probability of being reflected straight out. In such a cavity the radiation is in thermal equilibrium with the walls; each is at temperature T.

Classical (i.e. non-quantum) statistical mechanics shows that in thermal equilibrium each 'degree of freedom', such as motion in a given direction, or rotation about a given axis, has associated with it an average energy $\frac{1}{2}kT$, where k is Boltzmann's constant, and T is the absolute temperature. For example, an atom in three dimensions would have average energy $\frac{3}{2}kT$. Rayleigh (1842–1919) applied this result, called 'equipartition', to thermal radiation in 1900. It gives the energy density
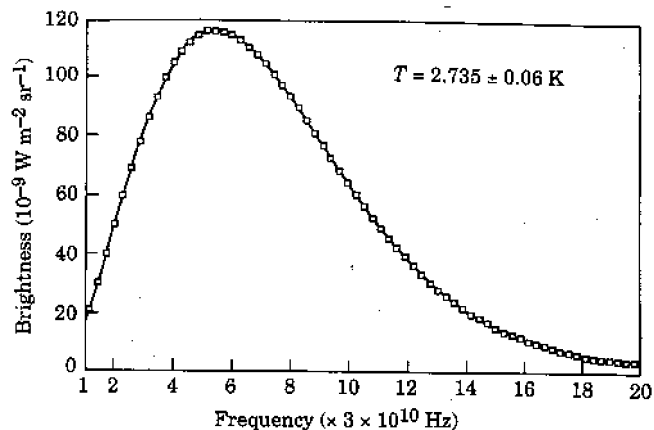
$$u(f,T) = \frac{8\pi f^2}{c^3} kT \tag{4}$$

This cannot be true for all frequencies: the energy per unit volume in the frequency range df is udf, and the total energy $\int_0^\infty udf$ would be infinite. However, the Rayleigh-Jeans law, as it is known (Jeans corrected the numerical multiplier in (4)), agreed with experiment at low frequencies.

Max Planck found, in October 1900, an ingenious interpolation between (4) and a high-frequency form due to Wilhelm Wien. Planck's formula for the energy density of thermal radiation is

$$u(f,T) = \frac{8\pi h}{c^3} \frac{f^3}{e^{hf/kT} - 1} \tag{5}$$

The new constant h (now 'Planck's constant') could be found by fitting (5) to the latest experimental data. The fit was within experimental error. By December 1900, 'after some of the most intense work in my life', Planck had a theoretical justification for his formula. In what he later described as 'an act of desperation', he assumed that the oscillators forming the walls of his idealised enclosure could only have energies which were integral multiples of hf, where f was the frequency of the oscillator.

Planck's formula gives the Rayleigh-Jeans law when $kT >> hf$, and the Wien (1893) law in the opposite limit. It leads easily to the Stefan-Boltzmann law that the total radiation energy is proportional to T^4. Further, the fit to the data also gave Boltzmann's constant k, and thus Avogadro's number. More information followed: the charge on the electron, for example, from Avogadro's number and electrochemical experiments. A triumph, but deeply unsettling. As Einstein was later to write 'All attempts to adapt the theoretical foundations of physics to these new notions failed completely. It was as if the ground had been pulled out from under one with no firm foundation to be seen anywhere, upon which one could have built'.

Heat capacities

In a monatomic gas the average kinetic energy is $\frac{3}{2}kT$ by the equipartition law. The total energy for N atoms is $E = \frac{3}{2}NkT$. The heat capacity is defined as the rate of change of energy with temperature, $C_V = (\partial E / \partial T)_V$ (it matters whether the change is at constant volume or constant pressure, hence the subscript V). Thus for a monatomic gas $C_V = \frac{3}{2}Nk$. This was found to be in agreement with experiment. But equipartition failed for molecular gases: a molecule has rotational and vibrational degrees of freedom, as well as translational ones, and some of these were apparently not excited. There was also a problem with solids: each atom can vibrate about its equilibrium position, so it has six degrees of freedom associated with its kinetic and potential energies. A heat capacity of $3Nk$ is expected, and indeed found, at room temperatures for most solids, but not for some (such as diamond), and not at low temperatures, where all heat capacities approach zero.

Again a fundamental mystery, to be resolved later by quantum mechanics. The explanation, first formulated by Einstein as we shall see, lies in the discrete nature of the energy levels of a quantum system. Some of the degrees of freedom are 'frozen' because there is not enough thermal energy to excite the system from its lowest energy level (the 'ground state') to higher energy levels.

The light quantum

We now come to Einstein's first quantum paper [4], written in March 1905. He begins: 'A profound formal distinction exists between the theoretical concepts which physicists have formed

regarding gases and other ponderable bodies and the Maxwellian theory of electromagnetic processes in so-called empty space'. That is, we talk of particles in mechanics, and of fields in electromagnetism, and these concepts are very different. He goes on to propose that light has a particle-like aspect: '... the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a *finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units*'. [my italics]

This is a revolutionary statement, quite different from Planck's assertion about the energies of his hypothetical oscillators. Planck had left the radiation as a field, unquantised. He and others were reluctant, to say the least, to accept the light quantum. As late as 1913, proposing Einstein for membership in the Prussian Academy, Planck, Nernst, Rubens and Warburg give the highest praise, concluding [6] 'That he may have sometimes missed the target in his speculations, as, for example, in his hypothesis of light quanta, cannot really be held too much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk'.

Einstein's paper begins by discussing black-body radiation. In particular, he reveals the similar form taken by the entropy of radiation and the entropy of an ideal gas (up to this point seen as waves and particles, respectively). Then he says 'If the entropy of monochromatic ratiation depends on volume as though the radiation were a discontinuous medium consisting of energy quanta of magnitude hf, the next obvious step is to investigate whether the laws of emission and transformation of light are also of such a nature that they can be interpreted or explained by considering light to consist of such energy quanta'. [I have changed Einstein's notation for h to the modern form: in this paper Einstein did not use Planck's constant h explicitly.] He goes on to show that Stoke's Rule, which says that monochromatic light is transformed through photoluminescence to light of a lower frequency, follows immediately from the assumption of light coming in and going out as quanta with energies hf_1 and hf_2, energy conservation implying $f_1 \geq f_2$. He notes that amount of fluorescent light should be proportional to the amount of incident light, and that 'there will be no lower limit for the intensity of incident light necessary to excite the fluorescent effect'.

The next section, titled 'Concerning the emission of cathode rays through the illumination of solid bodies', is on what is now known as the *photoelectric effect*. His picture of the process is as follows: 'Energy quanta penetrate into the surface layer of the body, and their energy is transformed, at least in part, into kinetic energy of electrons. The simplest way to imagine this is that a light quantum delivers its entire energy to a single electron; we shall assume that this is what happens ... Furthermore, we shall assume that in leaving the body each electron must perform an amount of work W characteristic of the substance. The ejected electrons leaving the body with the largest normal velocity will be those that were directly at the surface. The kinetic energy of

such electrons is given by $hf - W$.' [Again, the notation has been changed to use h.] This is Einstein's photoelectric equation:

$$K = hf - W \qquad (6)$$

Einstein continues 'If the body is charged to a positive potential V and is surrounded by conductors at zero potential, and V is just large enough to prevent loss of electricity by the body, it follows that $eV = hf - W$, where e denotes the electronic charge ... If the derived formula is correct, then V, when represented in Cartesian coordinates as a function of the frequency of the incident light, must be a straight line whose slope is independent of the nature of the emitting substance.' He goes on 'If each energy quantum of the incident light ... delivers its energy to electrons, then the velocity distribution of the ejected electrons will be independent of the intensity of the incident light; on the other hand, the number of electrons will ... be proportional to the intensity of the incident light.'

These predictions were to be completely verified by a series of experiments, notably Robert Millikan's [7] in 1916. But having vindicated Einstein's photoelectric theory, and extracted Planck's constant h as the slope of the eV versus frequency plot (see above), Millikan goes on to damn Einstein's idea of a light corpuscle, not with faint praise, but as follows: 'This hypothesis may well be called reckless first because an electromagnetic disturbance which remains localized in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of thoroughly established facts of interference'. However, it was Millikan's experiments which finally made Einstein's work acceptable to the Nobel Committee: he got the Nobel Prize in Physics in 1921 'for services to theoretical physics, and especially for his discovery of the law of the photoelectric effect". (Wilhelm Ostwald and others had three times nominated Einstein for the Nobel, starting in 1909, but the nomination failed because the topic (relativity) was too controversial.) Millikan himself got a Nobel in 1923, 'for his work on the elementary charge of electricity and on the photoelectric effect'.

Further striking confirmation of the light-quantum idea was provided in 1923 by Compton's paper on 'A quantum theory of the scattering of X-rays by light elements' [8]. Compton worked out the kinematics of the scattering of X-ray quanta by electrons. The electrons were given relativistic energy and momentum, the X-ray quanta had energy hf and momentum hf/c, as per Einstein's 1905 [4] and 1917 [9] papers, respectively. Conservation of energy and momentum gives the change in the wavelength of the X-ray:

$$\lambda_\theta = \lambda_0 + \frac{h}{mc}(1 - \cos\theta) \qquad (7)$$

(λ_0 is the wavelength of the incident X-ray, λ_θ that of an X-ray scattered through an angle θ; m is the electron mass).

Compton's paper contains both theory and experiment, in 'very satisfactory agreement', altogether a vindication of Einstein's relativity as well as of his light quantum concept. It is remarkable that, of the 26 references (11 of which are to Compton's own works), not one is to Einstein. Nor is Einstein mentioned by name, in the text or in the footnotes! Whether

this is because of personal antipathy, or careful positioning for the Nobel, I do not know. (The paper begins 'The hypothesis is suggested that when an X-ray is scattered it spends all of its energy and momentum upon some particular electron'. Hardly a new hypothesis, given Einstein's 1905 and 1917 papers [4] and [9].) In 1927 Compton was awarded the Nobel Prize 'for his discovery of the effect named after him'; it was shared with C.T.R. Wilson, of cloud-chamber fame.

Einstein's other quantum papers

Einstein made more than 20 further contributions to quantum theory. Only some will be discussed here. I give the titles in English; the journal and pages refer to the original paper. Lanczos [10] gives a summary of all of Einstein's papers in the period 1905 to 1915, and translations may be found in [2].

We mentioned, in Section 3(c), the specific heat mystery. Just two years after his light quantum, Einstein extends the quantum idea to vibrations in solids [12]. He assumes that atoms, held in place by interactions with their neighbours in the solid, vibrate at a single frequency f. Then Planck's assumption that the possible energies are integral multiples of $\varepsilon = hf$ gives the average energy of the oscillator (at temperature T):

$$\overline{\varepsilon} = \frac{\sum_{n=1}^{\infty} n\varepsilon e^{-n\varepsilon/kT}}{\sum_{n=0}^{\infty} e^{-n\varepsilon/kT}} = \frac{\varepsilon}{e^{\varepsilon/kT}-1} \tag{8}$$

For N atoms vibrating in 3 directions the average energy is $3N\overline{\varepsilon}$, and the heat capacity is

$$C = 3N\frac{\partial\overline{\varepsilon}}{\partial T} = 3Nk\left(\frac{\varepsilon}{kT}\right)^2 \frac{e^{\varepsilon/kT}}{(e^{\varepsilon/kT}-1)^2} \tag{9}$$

(If there are many vibration frequencies, one sums over terms like (9).) Einstein notes that C goes to zero for $kT \ll \varepsilon$, and to the high-temperature value $3Nk$ in the opposite limit. Not only these limits, but also the qualitative behaviour, are in agreement with experiment, as Einstein shows by comparing theory with the data for diamond. Einstein's note about sums over many frequencies was prescient: in fact one needs to consider the collective vibrations (now called phonons), as was done by Born and Karman and by Debye, in 1912.

The next two quantum papers of Einstein are reviews of the structure of radiation [13, 14]. In [13] he evaluates the energy fluctuations in thermal light, and finds the sum of two terms, one following from the corpuscular, the other from the wave nature of light. One can interpret the first as arising from the localised nature of the light quanta, the other from their interference as waves. Einstein also remarks (in [13]) on the fact that h and e^2/c have the same dimension, and thus their ratio may possibly be explained on the basis of pure numbers. We wish! The ratio $e^2/\hbar c \approx 1/137.036$, now known as the fine structure constant, and experimentally determined to parts per billion (namely 0.00729735253, with an uncertainty of ± 2 in the last place) is still eluding theoretical evaluation nearly a hundred years on.

We have already mentioned Einstein's 1917 paper 'On the quantum theory of radiation' [9], in relation to the photon momentum. It deserves more discussion, since in it Einstein shows that *stimulated* as well as *spontaneous* emission must exist, for radiation to be in stable equilibrium with a quantum system, e.g. a gas of molecules, which has discrete energy levels. He introduces the now famous A and B coefficients giving spontaneous and induced (or stimulated) transition probabilities: for energy levels 1 and 2, with $E_2 - E_1 = hf$, bathed in thermal radiation of energy density u, the rates for the transitions are postulated to be

$$1 \rightarrow 2 : BuN_1 ; \qquad 2 \rightarrow 1 : AN_2 + BuN_2 \tag{10}$$

where N_1 and N_2 are the numbers of molecules in states 1 and 2, respectively. Note that the stimulated (induced) rate is proportional to the radiation energy density at the frequency f. In thermal equilibrium

$$N_2/N_1 = e^{-E_2/kT}\big/e^{-E_1/kT} = e^{-hf/kT} \tag{11}$$

and also the transition rates given in (10) must be equal. Together these give Planck's law,

$$u = \frac{A/B}{e^{hf/kT}-1} \tag{12}$$

Thus entered stimulated emission, the existence of which none had suspected to this point; the application to masers (1954) and lasers (1961) came much later. (The term LASER comes from Light Amplification by *Stimulated* Emission of Radiation.)

In the same 1917 paper, Einstein considers the momentum interchange between the gas and the radiation field. He shows that light quanta of energy ε have to carry momentum ε/c for the velocity distribution in the gas to be Maxwellian. He writes '... we arrive at a consistent theory only if each elementary process is completely directional', meaning that the photons do not radiate as spherical waves from each molecule, as was thought to be the case. (Such spherical waves have zero net momentum.) Further, 'If a ray of light causes a molecule hit by it to absorb or emit ... an amount of energy hf in the form of radiation ... the momentum is always transferred to the molecule'. This is exactly the modern photon view of emission and absorption. And Einstein was pleased with the work: in a letter to his friend Michele Besso in September 1916 he writes 'With this, the existence of light quanta is practically assured'.

Let us summarise Einstein's achievements in quantum physics up to this point: (i) the light quantum, a kind of fusion of the wave and particle concepts, and its application to the photoelectric effect and other phenomena; (ii) the first quantum theory of specific heat; (iii) the fundamentals of the thermal equilibrium between matter and radiation, with the new concept of stimulated emission. Any one of these was substantial enough to warrant a Nobel Prize, and the first one did. (The special theory of relativity, and the explanation of Brownian motion, are also of Nobel standard!)

But Einstein had another mission: from 1911 he worked to generalise his relativity theory to include accelerated frames and gravitation. The General Theory was probably his greatest achievement, and its success in the prediction of the advance of

the perihelion of Mercury and the bending of light by a gravitational field made him world-famous. The General Theory (surely yet another Nobel) and then the unification of gravitation and electromagnetism, occupied him to the end of his life.

In the meantime the quantum revolution was continuing: we have already noted Bohr's model of the atom [5]. In 1924, Louis de Broglie (1892–1987) reasoned that if light had a particle nature, as Einstein had postulated, matter may have a wave nature. He assigned the 'de Broglie wavelength'

$$\lambda = h/p \tag{13}$$

to all particles (p is the momentum of the particle, λ the corresponding wavelength). Peter Debye (1884–1966) suggested to Erwin Schrödinger (1887–1961) that 'a wave needs a wave equation', and so the Schrödinger equation

$$H\psi = i\hbar \frac{\partial \psi}{\partial t} \tag{14}$$

was born. At the same time (1925) Heisenberg, Born and Jordan were developing matrix mechanics, in which observables such as position x and the corresponding momentum component p_x became non-commuting operators, with

$$xp_x - p_x x = i\hbar \tag{15}$$

Schrödinger later showed that wave mechanics and matrix mechanics were equivalent, and Paul Dirac (1902–1984) gave a relativistic generalisation, for electrons, of the Schrödinger equation. In two years, quantum theory had progressed to the point that most atomic and chemical properties of matter could be calculated, in principle. (In practice, the quantum many-body

problems, and divergences in quantum electrodynamics, proved decidedly non-trivial.)

As we mentioned, Einstein had other preoccupations. But he kept a sceptical interest in quantum physics, and in fact made one more fundamental extension of quantum theory, which led to a striking (and apparently crazy) prediction. In 1924 an Indian physicist, Satiendranath Bose (1894–1974), sent Einstein a manuscript for his opinion. Bose had derived the Planck formula by treating identical light quanta (i.e. those with the same energy, momentum and angular momentum) as *indistinguishable*. This implies new statistics (Bose statistics). Einstein translated the paper into German, and endorsed its publication [15], adding 'Bose's derivation of Planck's formula appears to me to be an important step forward. The method used here gives also the quantum theory of an ideal gas, as I shall show elsewhere'. Which he did [16], extending Bose's idea of indistinguishability to particles. Up to this time, identical particles, for example two helium atoms, were regarded as distinguishable, in principle at least. Their interchange was taken to give a new configuration; this plausible assumption led to wrong results in statistical mechanics (the Gibbs paradox, concerning the change in entropy when gases are allowed to mix, was one striking mismatch between theory and experiment). Einstein made the same assumption for particles as Bose had made for light quanta: interchange of two non-distinguishable particles has no effect. The Gibbs paradox disappeared, and quantum statistical mechanics was born. But there was more: Einstein realised that the new statistics implied that an ideal gas would form one enormous coherent quantum state at low enough

Louis de Broglie (1892–1987), Nobel 1929.

Max Born (1882–1970), Nobel 1954.

Erwin Schrödinger (1887–1961), Nobel 1933.

Werner Heisenberg (1901–1976), Nobel 1932.

temperatures. These 'Bose-Einstein condensates' were finally observed in 1995, seventy years after Einstein's prediction, in a triumph of sophisticated experimental techniques.

'Bose statistics', as they are now known, apply to particles with zero or integer spin, but not to spin ½ (i.e. intrinsic angular momentum $\hbar/2$), or $^3/_2$, $^5/_2$, etc. Enrico Fermi (1901–1954) and Paul Dirac saw independently in 1926 that half-integer spin particles are indistinguishable in a different way: when identical *fermions* are interchanged, the wavefunction ψ changes sign. (The probability density $|\psi|^2$ is unchanged, as it is when identical *bosons* are interchanged, when ψ remains ψ.) The elucidation of quantum statistics, and the prediction of what should be known as Einstein condensation (Bose did not consider particles at all, let alone predict their condensation at low temperatures) were Einstein's last great contributions to quantum theory, at the age of 45.

Epilogue

Albert and Mileva Marić parted in 1914, Mileva returning to Zurich with Hans Albert and Eduard. Einstein continued to support her and the boys during the war and after. They were divorced in February 1919, and he married his widowed cousin Elsa Löwenthal (née Einstein) in June of that year. Part of the divorce settlement was the assignment the money from a future Nobel Prize to Mileva! (The Nobel was finally awarded to Einstein in 1921.)

During and after the war Einstein made clear his pacifist views, and this, together with his Jewish background, and high visibility following the 1919 confirmation of his prediction of the bending of light, made him a target, well before Hitler took control of Germany. As Emilio Segrè (Nobel Prize 1959) writes in the second volume of his excellent history of physics [17] 'There was even an anti-Einstein scientific society where once respected and respectable names became mixed with demagogues, madmen, and future Nazi recruits ... The situation took an ugly turn, especially since the extremists would not hesitate to assassinate their enemies. The murder of W. Rathenau, Minister of Foreign Affairs of the Weimar Republic, was a warning of what could happen'. (Rathenau was a personal friend of Einstein.) With the arrival of Nazism, Einstein finally left Germany in December 1932, never to return.

He settled in Princeton, where he worked on the unification of electromagnetism and gravitation. Einstein maintained an interest in quantum mechanics, but believed it to be an incomplete description of reality. In his final quantum paper, with Podolsky and Rosen [18], the foundations of quantum mechanics are questioned. The authors assert that 'A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system'. This is the so-called *realist* viewpoint, that a quantum state actually has a certain property (spin along a certain axis, for example) prior to measurement. The *orthodox* quantum position is that the wavefunction ψ does not uniquely determine the outcome of a measurement: it provides (through $|\psi|^2$) only the statistical distribution of the possible results. The paper provides an

Einstein at Santa Barbara, 1933.
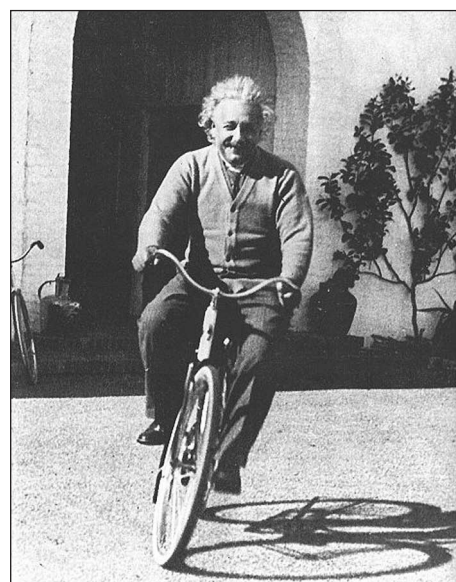
example, the 'EPR paradox', designed to show that the quantum position is absurd. The decay of a neutral pi meson into an electron and a positron, $\pi^0 \rightarrow e^- + e^+$, illustrates the point. In the rest frame of the pion, the electron and positron fly off in opposite directions. The pion has zero spin, so the conservation of angular momentum requires zero spin for the electron-positron pair. If the electron has spin up, the positron must have spin down, and vice versa. This is not a problem for the realist stance: the electron had spin up, we just did not know it (i.e. ψ does not contain the complete information, the EPR assertion). The orthodox position leads to a situation which is weird, but entirely confirmed by modern experiments: measurement of the electron spin *instantaneously* fixes the positron spin, *no matter how large the distance between the particle-antiparticle pair*! The measurements are perfectly correlated, by means of some 'spooky action-at-a-distance' (Einstein's words), called 'entanglement of the wavefunction' in modern quantum mechanics. The experiments verify perfect up/down correlation, at all distances. For Einstein, this meant propagation of some influence at infinite speed, contrary to relativity theory. But perfect correlation does not imply that information is being transmitted faster than light: the measurer of the electron spin has no influence on which result is obtained, and so cannot send a binary signal with elements 'up' and 'down'.

The EPR view led to the 'hidden variable' idea: if ψ gives an incomplete description of reality, there must be another variable, which, if we could calculate it, would give a complete description. Einstein died in 1955, at the age of 76. It was not till 1964 that J.S. Bell proved that *any* local hidden variable theory is incompatible with quantum mechanics. Thus nonlocality (spooky action-at-a-distance, or entanglement) seems to be with us, like it or not: experiment is king in science, and experiment has completely verified the orthodox view of quantum mechanics. Einstein is turning in his grave, or perhaps his soul is protesting strongly to the Maker of the Universe. No-one claims to *understand* quantum physics; we just know how to use it to calculate physical properties of radiation and matter. It works.

References

[1]   Winteler-Einstein, Maja. 1987. Albert Einstein – a biographical sketch. Pp. xv to xxii of the English translation of [2].

[2]   Stachel, John; Cassidy, Daniel C.; Schulman, Robert. (eds) 1987. *The Collected Papers of Albert Einstein. Vol. 1. The early years, 1879–1902*. Princeton Univ. Press, Princeton, New Jersey.

[3]   Hoffmann, Banesh; Dukas, Helen. 1973. *Albert Einstein, creator and rebel*. P. 26. Hart-Davis, MacGibbon, New York.

[4]   Einstein, Albert. 1905. On a heuristic point of view concerning the production and transformation of light. *Annalen der Physik 17*: 132–148 [in German]. English translation by Arons, A.B.; Peppard, M.B. 1965. *American Journal of Physics 33*: 367–374.

[5]   Bohr, Niels. 1913. On the constitution of atoms and molecules. *Philosophical Magazine 26*: 1–25.

[6]   Pais, Abraham. 1982. *'Subtle is the Lord …', the science and life of Albert Einstein*. Oxford Univ. Press, Oxford.

[7]   Millikan, Robert. 1916. A direct photoelectric determination of Planck's *h*. *Physical Review 7*: 355–388.

[8]   Compton, Arthur. 1923. A quantum theory of the scattering of X-rays by light elements. *Physical Review 21*: 483–502.

[9]   Einstein, Albert. 1917. On the quantum theory of radiation. *Physikalische Zeitschrift 18*: 121–128 [in German]. English translation in Barnes, Frank S. (ed.) 1972. *Laser Theory*. IEEE Press, New York.

[10] Cornelius Lanczos, Cornelius. 1974. *The Einstein Decade (1905-1915*. Elek Science, London.

[11] Einstein, Albert. 1906. Theory of light emission and absorption. *Annalen der Physik 19*: 371–379.

[12] Einstein, Albert. 1907. Planck's theory of radiation and the theory of specific heat. *Annalen der Physik 22*: 180–190.

[13] Einstein, Albert. 1909. On the present status of the radiation problem. *Physikalische Zeitschrift 10*: 185–193.

[14] Einstein, Albert. 1909. Evolution of our ideas concerning the essence and structure of radiation. *Physikalische Zeitschrift 10*: 817–826.

[15] Bose, Satiendranath. 1924. Planck's Law and the light quantum hypothesis. *Zeitschrift für Physik 26*: 178–181.

[16] Einstein, Albert. 1924, 1925. Quantum theory of monatomic ideal gases [in German]. *Sitzungsberichte der Preussischen Akademie der Wissenshaften 1924*: 261–267; *1925*: 3–14.

[17] Segrè, Emilio. 1980. *From X-rays to Quarks: modern physicists and their discoveries*. Freeman, San Francisco.

[18] Einstein, A.; Podolsky, B.; Rosen, N. 1935. Can quantum-mechanical description of physical reality be considered complete? *Physical Review 47*: 777–780.

Author's note

Einstein's light quanta have energy ε and momentum $p = \varepsilon/c$. They cannot be Lorentz-transformed to a rest frame; only particles that travel at less than light speed, and with momentum $p < \varepsilon/c$, have a rest frame. In 2003, I calculated the energy ε and momentum p of a localised pulse solution of Maxwell's equations, and found $p < \varepsilon/c$ [19]. These pulses can therefore be transformed to a zero-momentum frame (not a 'rest' frame: electromagnetic waves are never at rest, only particles with mass can be). I wrote 'We suspect that this will hold for all localised pulses of finite energy, since these must converge or diverge to some extent. If the same is true for photons, the Einstein picture of light quanta will need modification'. An anonymous referee remarked gently 'It may be my own prejudice, but I am cautious of work purporting to show that some aspects of Einstein's physics need modification'. Indeed! I brashly left the sentence intact.

By late 2003, I was able to show that all localised free-space pulse solutions of Maxwell's equation do have $\varepsilon > cp$ [20]. Thus a Lorentz boost at speed $c^2 p/\varepsilon$ will transform such pulses to their zero-momentum frame, 'in contrast to the Einstein light quanta, for which a zero-momentum frame does not exist'. My present view of the meaning of the $\varepsilon > cp$ theorem is that a semi-classical picture of the photon, based on localised solutions of Maxwell's equations, is not possible. Einstein's light quantum remains a very mysterious entity.

[19] Lekner, J. 2003. Electromagnetic pulses which have a zero momentum frame. *Journal of Optics A: Pure and Applied Optics 5*: L15–L18.

[20] Lekner, J. 2004. Energy and momentum of electromagnetic pulses. *Journal of Optics A: Pure and Applied Optics 6*: 146–147.