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WORKING PAPER 03/2016

July 2016

Working Papers in Public Finance



Chair in Public Finance
Victoria Business School

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The Elasticity of Taxable Income: Allowing for Endogeneity and Income Effects

John Creedy, Norman Gemmell and Josh Teng*

Abstract

This paper examines two problems in the estimation of the elasticity of taxable income. The first arises from the need to deal with endogeneity arising from the fact that the marginal tax rate and taxable income are jointly determined in a multi-tax structure. The approach taken in many empirical studies has been to use instrumental variable estimation. In contrast, the approach proposed in the present paper is to use ordinary least squares using proxy variables. It is shown that more robust, plausible results can be obtained using this approach. Secondly, the paper considers a potential role for income effects. One approach previously adopted has involved the addition of a term involving the proportional change in the net-of-tax average rate in addition to the change in the marginal net-of-tax rate. It is shown that the derivation of this specification, starting from the Slutsky equation, involves an invalid assumption (that virtual income can be neglected) at a crucial step. Nevertheless, for the New Zealand case, correction for this assumption leads to empirical results which also support the finding that income effects are negligible and statistically insignificant. In addition, the simpler specification can be derived more easily from a direct utility function. Following Kleven and Schultz (2014), income effects are also examined by introducing a term involving the proportional change in virtual income. Estimates reported here show a very small negative, but significant, coefficient on this variable when a proxy based on the expected tax rate is used, but a negligible and insignificant coefficient when a proxy based on an unchanged taxable income is used.

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1 Introduction

The ‘elasticity of taxable income’ (ETI) concept, first proposed by Feldstein (1995), captures the combined impact of various economic responses to changes in marginal income tax rates. The ETI measures the response of taxable income to variations in the net-of-tax rate, $1 - \tau$, rather than τ , and hence is expected to be positive. A key property of the elasticity is that it captures all responses to a change in the tax rate in a simple reduced-form specification and, under certain conditions, provides a convenient method of approximating the welfare effects of small tax changes; see, for example, Saez *et al.* (2012).

Many empirical estimates of this elasticity have been produced using a variety of methods applied to incomes before and after a tax reform, initially mainly for the US, but increasingly covering tax reforms in a wider range of countries. The majority of regression-based ETI studies make the explicit assumption that there are no income effects, using a specification in which there is a constant elasticity of taxable income with respect to $1 - \tau$. This provides a substantial simplification, particularly where welfare effects are of interest. Furthermore, the analysis of changes in tax revenue is simplified by the assumption that taxable incomes in a particular tax bracket do not respond to change in marginal tax rates in lower tax brackets, despite such changes having an effect on the average tax rate of those in the higher brackets.

Much emphasis has been placed on the appropriate method of dealing with endogeneity, arising from the fact that in a multi-rate tax function the tax rate and taxable income are jointly determined, and modelling the (potentially large) extent of income changes that have nothing to do with tax changes. In tackling the former problem, the literature has almost exclusively relied on instrumental variable estimation. Typically, various forms of lagged taxable incomes have been used in an attempt to capture non-tax-related income movements.

The present paper has two main aims. The first aim relates to the method of estimation used. A particular proxy variable for the post-reform tax rate, rather than applying instrumental variables, is proposed. The proxy tax rates are based on counterfactual post-reform incomes, that is the incomes – estimated using a specified dynamic process – that would have eventuated in the absence of a tax reform. The second aim of the paper is to examine a possible role for income effects. In attempting to test for income effects, several studies have taken as their starting point the basic components of the fundamental Slutsky equation, involving income and substitution (compensated) effects, along with the basic expression for the total change in taxable income resulting from marginal tax rate changes and (virtual) income changes. These are then manipulated to obtain an expression for the

proportional change in taxable income in terms of changes in both the net-of-tax marginal rate, $1 - \tau$, and the net-of-tax average rate, $1 - ATR$; see, for example, Gruber and Saez (2002), Bakos *et al.* (2008), and Gottfried and Witczak (2009).¹ The derivation is examined in detail, and an alternative rationale for their specification is presented.

A different approach is taken by Kleven and Schultz (2014), who begin from a more general utility-maximising framework in which income and substitution effects are identifiable respectively from variables measuring virtual income and the net-of-tax marginal rate. Their empirical specification involves changes in the logarithm of virtual income, rather than $1 - ATR$, based on a linearisation of their theoretical model. The use of virtual income is also examined here.

Section 2 discusses the question of endogeneity and suggests the use of an exogenous proxy variable defined as the expected tax rate, for each individual, based on the individual's projected conditional distribution of income in the post-reform year, given income in two previous years. Comparisons using regression methods based on both instrumental variables and proxy variables are made in Section 3. In obtaining these and further estimates reported below, data relating to the New Zealand income tax reform of 2001 are used. Section 4 concentrates on the role of income effects using a specification involving the net-of-tax average rate, and Section 5 considers a specification in terms of virtual income. Conclusions are in Section 6.

2 Dealing with Endogeneity

In estimating the ETI, a constant elasticity form is ubiquitous in the literature, whereby the logarithm of an individual's taxable income, $\log y$, is expressed as a linear function of the logarithm of the net-of-tax rate, $\log(1 - \tau)$, facing the individual. Fixed effects are generally eliminated by taking first-differences, giving a specification that takes the form:

$$\Delta \log y = \alpha + \eta \Delta \log(1 - \tau) + \text{control variables} + u \quad (1)$$

where u is a random variable and η is the elasticity of taxable income. This explicitly assumes that income effects of the tax rate change are assumed to be absent, which is consistent with a quasi-linear utility function having a constant marginal utility of net income (consumption).² Inevitably there are income changes which would occur in the absence of tax changes. The challenge is thus to avoid attributing those exogenous income

¹Bach *et al.* (2011) also follow a similar approach, but in the context of income splitting for couples. They refer to the revenue-maximising top marginal rate as the 'optimal' rate.

²An explicit direct utility function is discussed below in Section 4.

changes to the tax rate changes, such that various forms of lagged income (and age terms) are typically prominent among the control variables in (1).

There is a well-known potential endogeneity problem when relating income changes to changes in the net-of-tax rate. With a nonlinear income tax structure, taxable income and the marginal tax rate (in the relevant tax bracket) are jointly determined. The usual approach is to use instrumental variable estimation and, following Gruber and Saez (2002), the net-of-tax rate instrument used extensively in the literature is the net-of-tax rate that would be applicable post-reform but with unchanged income levels.³

Recently, this ‘standard’ instrument has been challenged on a number of grounds, most significantly perhaps by Weber (2014) who demonstrates that the instrument is inconsistent under certain plausible assumptions about the income-generating process, leading to biased ETI estimates.⁴ In particular, she argues (and finds for her US tax reform dataset) that the standard instrument will not deal with endogeneity where there is serial correlation in the income generating process.

For New Zealand, Carey *et al.* (2015) examine annual taxable income dynamics over a period when there were no tax changes and demonstrate that the process exhibits considerable exogenous annual variability and is characterised by substantial mean reversion and serial correlation.⁵ Following Weber (2014) this could be expected to render the standard instrument biased. Carey *et al.* (2015) show that, for the 2001 tax change in New Zealand, this standard instrument is particularly weak. They construct alternative instruments designed to accommodate New Zealand’s volatile income dynamics and which can capture the counterfactual income dynamics. The parameters describing the ‘no reform’ relative income dynamics are assumed to be applicable to the reform years had there been no reform, and are used to construct post-reform incomes that are expected in the absence of reform. Their favoured instrument is based on the full conditional distribution of income for each taxpayer, given pre-reform income, which is used to obtain an expected *tax rate*.

³Studies using this instrument include, for example, Moffitt and Wilhelm (1998), Auten and Carroll (1999), Goolsbee (2000), Sillamaa and Veall (2000), Aarbu and Thoresen (2001), Gruber and Saez (2002), Selèn (2002), Giertz (2004, 2007, 2010), Hansson (2004), Kopczuk (2005), Auten *et al.* (2008), Heim (2009). Carroll (1998) suggests creating an instrument based on income for several sample years, and the Auten *et al.* (2008) instrument is based on the tax rate evaluated at the average taxable income over a seven year period. For New Zealand, Thomas also used income controls, such as taxable income in 1986, to deal with ‘reversion-to-the-mean’ effects.

⁴This instrument has also been questioned by, for example, Gelber (2010), Blomquist and Selin (2010), Auten and Kawano (2011), Weber (2012), Holmlund and Soderstrom (2011), and Burns and Ziliak (2012). Alternative approaches typically involve using income data for years prior to the tax change; see, for example, Burns and Ziliak (2012).

⁵See Creedy (1985) for an early application. Creedy (1995) and Moffitt and Gottschalk (2011) provide reviews of some of this literature.

Income dynamics are described as follows. Let μ_j denote the arithmetic mean of log-income in period j and $u_{j,i}$ is a random error term, where $E(u_{j,i}) = 0$ and $V(u_{j,i}) = \sigma_u^2$ are respectively the constant mean and variance of $u_{j,i}$ for all j :⁶

$$\log y_{j,i} - \mu_j = \alpha_2 (\log y_{j-1,i} - \mu_{j-1}) + \alpha_3 (\log y_{j-2,i} - \mu_{j-2}) + u_{j,i} \quad (2)$$

Ideally, α_2 and α_3 would be estimated from regressions using data for years prior to the tax changes. These could then be used to project hypothetical ‘no reform’ incomes forward to the relevant post-reform years. Given the parameter estimates, the computation of the expected post-reform tax rate for each individual, based on the counterfactual (that is, no tax change) distribution of income in the required year (conditional on incomes in two years before the tax change) is explained in Appendix A below.

The ubiquitous use of instrumental variable estimation in the ETI literature is a response to the fundamental problem that the variable of interest is unobservable. This is summarised by Saez *et al.* (2012, p. 18) as follows: ‘in order to isolate the effects of the net-of-tax rate, one would want to compare observed reported incomes after the tax rate change to the incomes that would have been reported had the tax change not taken place. Obviously, the latter are not observed and must be estimated’. Attempts to find an observable proxy for this unobserved counterfactual have, since Gruber and Saez (2001), involved using net-of-tax rates associated with *actual* incomes, which therefore require instrumenting to deal with the endogeneity created by nonlinear tax structures.⁷ But as Weber (2014) demonstrated, especially in the presence of income dynamics involving regression to the mean and serial correlation, it is difficult to identify suitable instruments based on actual incomes. For the US 1986 tax reform case, she finds some evidence that use of suitably long lags in incomes may help identify genuinely exogenous instruments.

Weber’s (2014) objective is of course to find more reliable exogenous instruments for the endogenous actual income variable in the second stage regression, where the latter variable is a proxy for the true counterfactual. By contrast, it can be argued that the use of estimates of income dynamics (obtained from years involving no tax changes) to produce, for each

⁶This autoregressive specification for *relative* incomes is consistent with a dynamic process involving regression towards the mean of ρ , where $\log y_{j,i} - \mu_j = \rho (\log y_{j-1,i} - \mu_{j-1}) + u_{i,j}$, and first-order serial correlation of γ , where $u_{i,j} = \gamma u_{i,j-1} + \varepsilon_{i,j}$. Creedy (1985) shows that, $\alpha_2 = \gamma + \rho$ and $\alpha_3 = -\gamma\rho$, such that estimates of ρ and γ may be obtained as $\rho = \left\{ \alpha_2 + (\alpha_2^2 + 4\alpha_3)^{0.5} \right\} / 2$ and $\gamma = \left\{ \alpha_2 - (\alpha_2^2 + 4\alpha_3)^{0.5} \right\} / 2$. In principle additional lags could be added but, in practice, previous studies have found only one or two lags to be sufficient to capture the dynamics of interest.

⁷Recent literature has discussed how to deal with transitory shock components of actual incomes. For discussion, see Weber (2014), who is the first to examine in detail the implications of serial correlation in this transitory component for IV approaches to estimation.

individual, an expected tax rate actually provides the required counterfactual post-reform incomes that *by construction* are exogenous to the tax reform. Hence, this approach can provide a direct proxy for the unobservable incomes which would ideally be used to construct the counterfactual net-of-tax rate to be included on the right-hand-side of taxable income regressions.⁸ There is therefore no need to invoke the instrumental variable (IV) approach. Indeed, in this case the use of IV methods, rather than ordinary least squares (OLS) with the proxy expected tax rate, risks introducing greater inefficiency into the estimates without the compensation of consistency gains or reduced bias. Comparisons are made in the following section.

3 Comparisons Using IV and Proxy Variables

This section reports comparisons of results obtained using instrumental variables and proxy variables, based on the 2001 reforms in New Zealand.

3.1 The 2001 Tax Reform in New Zealand

Table 1 shows the pre- and post-reform New Zealand tax rates. The tax rate changes took effect in the 2001 tax year (April 2000 to March 2001). After a few years of minor tax changes, the 2001 reforms represented a significant policy change, involving a number of tax rate changes, especially the introduction of a new top marginal rate of 0.39 above \$60,000. The announcement of the tax changes led to a certain amount of income shifting between periods, so that a comparison between incomes in 2000 and those immediately after the change would give misleading results. Using a longer interval allows for these inter-temporal shifts in income to settle down. To exclude these anticipation effects, regressions below compare taxable incomes in 1999 and 2002.

Of the four marginal rates in the tax schedule in 1999, the reform involved 0.75 and 3 percentage point decreases in two middle tax rates respectively (from 21.75 and 24 per cent to a common 21 per cent rate) and a 6 percentage point increase in the top rate (from 33 to 39 per cent) for incomes above \$60,000.⁹ These represent approximate percentage changes in the three reformed tax rates (using log differences) of -3.5 , -13.4 and $+16.7$ per cent.¹⁰

⁸The compensating weakness of the ‘counterfactual income dynamics’ approach is the untestable assumption that systematic income dynamics observed over non-reform years would have applied during the reform years had there been no reform. The robustness of income dynamic parameter estimates based on a number of non-reform year episodes provides some evidence that this assumption is reasonable; see Appendix C.

⁹The lowest rate, applicable up to \$16,000, remained at 15 per cent, with the 33 per cent rate applicable to incomes in the range \$38-60,000.

¹⁰Equivalent percentage changes in the net-of-tax rate, $1 - \tau$, are -1.0 , -2.9 and $+9.4$ per cent.

Table 1: New Zealand Income Tax Structure: 1999 and 2002

1999 Tax Structure		2002 Tax Structure	
Income range	Tax rate	Income range	Tax rate
1 – 9,500	0.15	1 – 9,500	0.15
9,501 – 34,200	0.2175	9,501 – 34,200	0.21
34,201 – 38,000	0.24	34,201 – 38,000	0.21
> 38,001	0.33	38,001 – 60,000	0.33
		> 60,001	0.39

This makes the New Zealand reform a particularly helpful one to analyse in this context because of the mixture of tax rate increases and decreases (and no change) across a wide range of incomes.

Evidence from Claus *et al.* (2012), who examine taxpayer income share changes, suggests that responses to the 2001 tax reform did not persist into the 2003-05 period, suggesting that this may provide a suitable ‘no reform’ period for income dynamics estimation.¹¹ Hence parameters of equation (2) were estimated by regressing $\log y_{05,i} - \mu_{05}$ on $\log y_{04,i} - \mu_{04}$ and $\log y_{03,i} - \mu_{03}$, for the same individuals as used in the estimation of the elasticity of taxable income.

Regarding estimation of income dynamics parameters, in the years immediately prior to the 2001 tax reform, a number of minor changes in tax parameters make the years before reform less suitable as a basis for estimating the no-reform income dynamics. However, the tax structure remained unchanged for a number of years after the 2001 tax policy change.¹² Parameter were therefore obtained from running the specification in (2) on data for 2003-05.¹³ Resulting estimates of α_2 and α_3 are 0.668 and 0.199 respectively, with t -values of 145.5 and 43.4, and $\sigma_u = 2.6214$.¹⁴ These α values imply estimates of regression towards the mean of $\phi = 0.891$, and serial correlation of $\gamma = -0.223$. Hence the data suggest relatively rapid regression towards the mean along with negative serial correlation whereby, for example, those who experience a large income increase are more likely to have a subsequent decrease.

¹¹See Claus *et al.* (2002, Figures 1 and 2). There is a possibility that some individuals experience marginal tax rate changes resulting from fiscal drag. But with low inflation, the vast majority of income changes over this period can reasonably be thought to reflect non-tax related income movements.

¹²Though subsequent substantive personal income tax changes did not occur until 2008, a three percentage point reduction in the corporate tax rate in 2006 may have generated responses by personal income taxpayers. Shifting between personal and corporate tax status is relatively easy in New Zealand.

¹³In fact, as Appendix C shows, estimates of the regression to the mean, and serial correlation, parameters (ϕ and γ respectively), are very similar when estimated over alternative three-year periods prior to, and after, the 2000-01 reforms despite some minor tax changes in some of those years.

¹⁴The mean of logarithms of income in 2003, 2004 and 2005 are 10.311, 10.367 and 10.367, with standard deviation of logarithms of 0.9194, 0.9110 and 0.9651 respectively.

The database used here was constructed by randomly sampling the Inland Revenue Department’s individual taxpayer population, and covers the period 1994–2009. The number of taxpayers in the random sample rises from 138,464 in 1999 to 139,420 in 2002. The sample is weighted to match the individual taxpayer population, which increased from 2,800,528 taxpayers in 1999 to 2,962,200 in 2002. The database is not constructed on a household basis. It contains welfare benefits data administered to individual taxpayers and family assistance provided to a nominated parent but not both parents.

3.2 Results Using Instrumental Variables

Based on the New Zealand tax reform described in the previous subsection, Carey *et al.* (2015) examined taxable income responses based on the ETI specification in (1) and the income dynamics specification in (2).¹⁵ In addition to age and income terms among the control variables in the regression, a dummy variable was included to capture the composition of individuals’ incomes, specifically whether they received only wage or salary income in the pre- and post-reform years.¹⁶

For the regressions, several restrictions were imposed on the data. Age restrictions were imposed in order to remove those taxpayers likely to be in the very early stages of their careers as well as those becoming eligible for New Zealand superannuation. Only taxpayers aged 25-64 across the entire period are included. Income restrictions are also imposed, in order to remove very high income earners (over \$1 million in 1999) and low-income earners under \$16,000. The latter face benefit abatement rates which mean that their effective marginal tax rates differ significantly from those of a standard taxpayer. Finally, those without sufficient income data across all relevant years (1998, 1999, 2002, 2003, 2004 and 2005) are necessarily excluded. As a result, the sample size is reduced to 38,744, which, when weighted up to reflect the population, represents 803,920 individual taxpayers. Further details of the data, the restrictions and the sampling process are given in Appendix B.

Table 2 reports regression results without allowing for income effects, using the frequently-used ‘standard’ instrument based on the post-reform tax rate applied to an unchanged income and the ‘expected tax rate’ described above and in Appendix A: these are taken from Carey *et al.* (2015). Each regression takes the constant elasticity form and includes, in addition to the relevant instrument, terms in age and age-squared, log income in 1999,

¹⁵In regressions on (1), 2002 is treated as period t and 1999 (1998) as period $t - 1$ ($t - 2$).

¹⁶The dummy was set equal to 1 if the individual received, either in addition to or instead of wage and salary income, any ‘other income’ in 1998, 1999 and 2002. Other income includes: dividends, trust and estate income, partnership, rental income, business or other income, shareholder employee income, and overseas income.

the lagged change in log income and the ‘other income’ dummy (equals one if the taxpayer has non-wage and salary income). The poor performance of the instrument based on an unchanged income – a large negative elasticity that is not significantly different from zero – is discussed in detail in Carey *et al.* (2015). The results in Table 2 show the dramatic improvement when using the expected tax rate instrument.¹⁷

Table 2: Regression Estimates using Alternative Instruments

Dependent variable: $\log y_{02} - \log y_{99}$		
Independent variables	Parameter	
	estimate	<i>t</i> -value
<i>Instrument based on unchanged income</i>		
Intercept	26.70378	0.11
$\Delta \log(1 - \tau)$	-175.027	-0.11
Age	-0.23974	-0.09
Age-squared	0.0044	0.10
$\log y_{99}$	-2.59084	-0.12
$\log y_{99} - \log y_{98}$	1.896705	0.10
Other income dummy	-2.31202	-0.11
Adjusted $R^2 = 0.00007$; $N = 38,744$		
<i>Instrument based on expected tax rate</i>		
Intercept	1.244	14.3
$\Delta \log(1 - \tau)$	0.676	5.4
Age	0.036	13.1
Age-squared	-0.0005	-15.0
$\log y_{99}$	-0.179	-26.3
$\log y_{99} - \log y_{98}$	-0.123	-17.3
Other income dummy	0.046	6.5
Adjusted $R^2 = 0.039$; $N = 38,744$		

3.3 Results Using Proxy Variables

This subsection reports OLS regressions using the constructed counterfactual net-of-tax rate based on the income dynamics described above as a proxy variable. It could also be argued that the ‘standard instrument’ – the post-reform net-of-tax rate associated with pre-reform income – is merely a simple form of counterfactual income dynamics; that is, where incomes at t are expected to remain unchanged from incomes at $t - 1$ in the absence of reform. It is therefore of interest to compare how this variable performs when treated as a counterfactual

¹⁷Carey *et al.* (2015) show that the expected tax rate satisfies established diagnostic tests for a valid instrument, while the ‘standard instrument’ does not.

proxy rather than as an instrument, though of course this proxy variable must be recognised as likely to be potentially endogenous for the reasons articulated by Weber (2014) and others. Nevertheless, its weakness *as an instrument* is avoided by including it directly in regressions as a proxy variable.

Table 3: Regression Estimates using Alternative Proxies

Dependent variable: $\log y_{02} - \log y_{99}$		
Independent variables	Parameter estimate	<i>t</i> -value
<i>Tax rate proxy based on rate for unchanged income</i>		
Intercept	1.17163	11.39
$\Delta \log(1 - \tau)$	0.31184	2.63
Age	0.03516	13.64
Age-squared	-0.0004634	-15.47
$\log y_{99}$	-0.17237	-19.83
$\log y_{99} - \log y_{98}$	-0.11360	-17.52
Other income dummy	0.03798	5.92
Adjusted $R^2 = 0.0429$; $N = 38,744$		
<i>Tax rate proxy based on expected tax rate</i>		
Intercept	1.70846	16.71
$\Delta \log(1 - \tau)$	0.37524	5.75
Age	0.03455	13.40
Age-squared	-0.0004566	-15.24
$\log y_{99}$	-0.22202	-26.05
$\log y_{99} - \log y_{98}$	-0.1146	-17.68
Other income dummy	0.03644	5.70
Adjusted $R^2 = 0.0435$; $N = 38,744$		

The results of carrying out OLS regressions with these proxy variables on the same dataset as that used for the Table 2 regressions, are shown in Table 3. The most obvious difference from Table 2 is that the parameter estimate on the ‘standard instrument’, when used as a proxy variable, is more plausible than when used as an instrument: the estimate of the elasticity of taxable income is significantly different from zero and has a ‘sensible’ sign and order of magnitude. Of course, it could be argued that this simple proxy is endogenous, for the reasons already elaborated. However, the OLS estimate of the ETI using this ‘standard proxy’ is similar to that obtained when the more clearly exogenous ‘expected tax rate proxy’ is used, as shown in the lower half of the table; the latter generally gives rise to higher *t*-values, especially for $\Delta \log(1 - \tau)$. Also, the elasticity estimate obtained from this exogenous proxy is smaller than in Table 2 (0.375 versus 0.676), the difference being

marginally significant at the 5% level.

These results would seem to support the *a priori* case made above for the use of proxy variables, where suitable longitudinal ‘no reform’ income dynamic information is available. All the results reported in the following sections are therefore based on the application of ordinary least squares using proxy variables, rather than instrumental variables.¹⁸

4 Income Effects using Average Tax Rates

As is well-known, the ETI estimates obtained from a specification in (1) assume that there are no income effects, but if such effects are present then ETI estimates may suffer from standard omitted variable bias.¹⁹ In testing for income effects, this section examines the approach used to justify the use of a term involving (proportional) changes in the average net-of-tax rate, $1 - ATR$, in addition to changes in $1 - \tau$. It is necessary to delve into the analytics in some detail. Consider the general linear tax function:

$$T(y) = \tau y - R \quad (3)$$

where τ and R are respectively the marginal tax rate and virtual income. For a multi-rate structure this can be considered as applying to the relevant section, as shown below. If taxable income is a function of τ and R , the change in income resulting from a change in both variables is:

$$\begin{aligned} dy &= \frac{\partial y}{\partial \tau} d\tau + \frac{\partial y}{\partial R} dR \\ &= -\frac{\partial y}{\partial (1 - \tau)} d\tau + \frac{\partial y}{\partial R} dR \end{aligned} \quad (4)$$

Gruber and Saez (2002) and Bakos *et al.* (2008) make use of Slutsky’s Theorem. In general terms, for a consumer with total expenditure, m , consider a good with uncompensated (or Marshallian) demand of x , price p , income elasticity, e , budget share of w , and compensated and uncompensated own-price elasticities of η and σ respectively, the Slutsky Theorem states that:

$$\sigma = \eta + ew \quad (5)$$

and:

$$ew = \left(\frac{px}{m}\right) \left(\frac{\partial x}{\partial m} \frac{m}{x}\right) = p \frac{\partial x}{\partial m} \quad (6)$$

¹⁸Results were nevertheless also obtained using instrumental variable methods. However, these performed badly, producing no consistency and often with wrong signs suggesting that the loss of efficiency may be severe in this case. This is plausible given the known volatility in the New Zealand annual income data.

¹⁹Relatively few ETI studies have tested formally for income effects, among them Gruber and Saez (2002) and Kleven and Schultz (2014), who each conclude that they can be ignored for ETI estimation purposes.

In the present context the ‘price’ is the ‘net-of-tax’ marginal rate, $1 - \tau$, and ‘consumption’ is taxable income, y , while ‘income’ is measured by ‘virtual income’, R . Hence, denoting the compensated elasticity of taxable income (ETI) as β^* , and the income effect (ew) as θ , these are given by:

$$\beta^* = \left(\frac{1 - \tau}{y} \right) \frac{\partial y}{\partial (1 - \tau)} \quad (7)$$

and:

$$\theta = (1 - \tau) \frac{\partial y}{\partial R} \quad (8)$$

Hence, using (5), the uncompensated ETI, denoted β , is simply:

$$\beta = \beta^* - \theta \quad (9)$$

In what follows these are considered to be fixed: this contrasts with an assumption that parameters of a utility function are fixed, as discussed below. From (7):

$$-\frac{\partial y}{\partial (1 - \tau)} d\tau = -\frac{\beta^* y}{1 - \tau} d\tau \quad (10)$$

and from (8):

$$\frac{\partial y}{\partial R} dR = \frac{\theta}{1 - \tau} dR \quad (11)$$

Then using $\beta^* = \beta + \theta$, (4) becomes:

$$dy = -\frac{(\beta + \theta)y}{1 - \tau} d\tau + \frac{\theta}{1 - \tau} dR \quad (12)$$

and:

$$\begin{aligned} \frac{dy}{y} &= -\beta \frac{d\tau}{1 - \tau} - \theta \left\{ \frac{yd\tau - dR}{(1 - \tau)y} \right\} \\ &= \beta \frac{d(1 - \tau)}{1 - \tau} + \theta \left\{ \frac{dR - yd\tau}{y(1 - \tau)} \right\} \end{aligned} \quad (13)$$

Equation (13) is readily converted to changes in logarithms, using $d \log y = dy/y$ and, for the first term in (13), $d \log(1 - \tau) = d(1 - \tau)/(1 - \tau)$. The conversion of this expression into one involving the average tax rate is discussed in the following subsection.

4.1 Marginal and Average Rate Changes

To consider the role of the average tax rate, ATR , consider again the linear form (for a given marginal rate and segment of the tax function) in (27). This is given by:

$$ATR = \frac{y\tau - R}{y}; \quad (14)$$

hence:

$$1 - ATR = \frac{R + y - \tau y}{y} \quad (15)$$

Using the fact that, in general, $d \log \frac{x}{z} = d \log x - d \log z = \frac{dx}{x} - \frac{dz}{z}$, log-changes in (15) gives:

$$d \log (1 - ATR) = \frac{d(R + y - \tau y)}{R + y - \tau y} - \frac{dy}{y} \quad (16)$$

and:

$$d \log (1 - ATR) = \frac{dR - yd\tau + (1 - \tau) dy}{R + y(1 - \tau)} - \frac{dy}{y} \quad (17)$$

To see if this expression, involving log changes in the average net-of-tax rate corresponds in any way to the term in curly brackets in (13), the general result that:

$$\frac{a}{x + b} = \frac{a}{b} \left(1 - \frac{x}{x + b} \right) \quad (18)$$

can be used to rewrite the first term on the right hand side of (17) as:

$$\frac{dR - yd\tau + (1 - \tau) dy}{R + y(1 - \tau)} = \frac{dR - yd\tau + (1 - \tau) dy}{y(1 - \tau)} \left\{ \frac{(1 - \tau) y}{R + y(1 - \tau)} \right\} \quad (19)$$

Hence:

$$d \log (1 - ATR) = \frac{dR - yd\tau}{y(1 - \tau)} \left\{ \frac{(1 - \tau) y}{R + y(1 - \tau)} \right\} - \left\{ \frac{R}{R + y(1 - \tau)} \right\} \frac{dy}{y} \quad (20)$$

Thus the term in curly brackets in (13) becomes:

$$\frac{dR - yd\tau}{y(1 - \tau)} = \left\{ \frac{R + y(1 - \tau)}{(1 - \tau) y} \right\} d \log (1 - ATR) + \left\{ \frac{R}{(1 - \tau) y} \right\} \frac{dy}{y} \quad (21)$$

Substituting these results into (13) finally gives, in terms of log changes:

$$\left\{ 1 - \frac{\theta R}{(1 - \tau) y} \right\} d \log y = \beta d \log (1 - \tau) + \theta \left\{ \frac{R + y(1 - \tau)}{(1 - \tau) y} \right\} d \log (1 - ATR) \quad (22)$$

Gruber and Saez (2002), Bakos *et al.* (2008)²⁰ and Gottfried and Witczak (2009) all make the crucial assumption that R can be neglected in (22), which immediately leads to the convenient form:

$$d \log y = \beta d \log (1 - \tau) + \theta d \log (1 - ATR) \quad (23)$$

²⁰In Bakos *et al.* (2008, p. 31), typographical errors mean that $y(1 - \tau) = y - \tau y + R$ appears as $y(1 - \pi) = y - y + R$, and in their following (unnumbered) equation, $d \left(\frac{R + y - \tau y}{y} \right)$ is printed instead of $d \log \left(\frac{R + y - \tau y}{y} \right)$.

However, this is unrealistic, as demonstrated below. Furthermore, it ignores the fact that making this assumption is effectively returning to the case where the marginal and average tax rates are equal. If a marginal tax rate in a lower tax bracket changes, this affects the average tax rate only via a consequent change in R : hence assuming R is negligible simply rules out that possibility. Yet, in empirical applications, the ATR appears to be calculated using the actual structure in which R plays an important role: hence R is ignored in one part, but not another, of the expression for $d \log y$.

To examine the term, R , in practice, consider a multi-step income tax schedule, defined by a set of income thresholds, a_k , for $k = 1, \dots, K$, and marginal income tax rates, τ_k , applying within tax brackets, that is between adjacent thresholds a_k and a_{k+1} .²¹ It can be written as:

$$\begin{aligned} T(y) &= \tau_1(y - a_1) & a_1 < y \leq a_2 \\ &= \tau_1(a_2 - a_1) + \tau_2(y - a_2) & a_2 < y \leq a_3 \end{aligned} \quad (24)$$

and so on. If y falls into the k th tax bracket, so that $a_k < y \leq a_{k+1}$, $T(y)$ can be rewritten for $k \geq 2$ as:

$$T(y) = \tau_k(y - a_k) + \sum_{j=1}^{k-1} \tau_j(a_{j+1} - a_j) \quad (25)$$

Letting $b_k = \sum_{j=1}^{k-1} \tau_j(a_{j+1} - a_j)$ this becomes:

$$\begin{aligned} T(y) &= \tau_k(y - a_k) + b_k \\ &= \tau_k y - (a_k \tau_k - b_k) \end{aligned} \quad (26)$$

The tax function facing an individual whose income, y , falls into the k th tax bracket can therefore be described by the appropriate linear segment (operating between thresholds) for that bracket:

$$T(y) = \tau_k y - R_k \quad (27)$$

where R_k is ‘virtual income’, defined as the implicit non-wage net income.²² Hence:

$$R_k = a_k \tau_k - b_k \quad (28)$$

The value of R_k will of course vary across taxpayers in different tax brackets and, for higher-bracket taxpayers, can be substantial. For example, consider the tax schedule facing single earners in New Zealand in 2010, before the major tax changes initiated that year. This had four tax brackets with lower income thresholds of 0, 14,000, 48,000 and 70,000, with

²¹If concentration is on higher-income brackets, it is not necessary to allow for benefits and associated abatement, or taper, rates, which apply mainly to lower-income groups.

²²Net income is $(1 - \tau_k)y + R_k$, so that virtual income is net income when $y = 0$.

marginal tax rates of 0.125, 0.21, 0.33 and 0.38 respectively. The equivalent values of R are 0, 1,190, 6,950 and 10,450 respectively. Hence, for incomes of, for example, 10,000, 20,000, 55,000 and 75,000, the values of R expressed as a percentage of $y(1 - \tau)$ are found to be 0, 7.5, 18.9 and 22.5 per cent respectively. In the bottom tax bracket the tax function is obviously proportional, so clearly $R = 0$.²³

Instead of neglecting R , rewrite (23), using $z = R/\{y(1 - \tau)\}$, as:

$$d \log y = \beta \left[\left\{ \frac{1}{1 - \theta z} \right\} d \log (1 - \tau) \right] + \theta \left[\left\{ \frac{1 + z}{1 - \theta z} \right\} d \log (1 - ATR) \right] \quad (29)$$

As an empirical specification to identify values of β and θ , this creates difficulties since it is nonlinear in both parameters.²⁴ A possible approach involves carrying out linear regressions of (29), in which the terms in square brackets are constructed for a range of imposed values of θ , say θ' . Resulting estimates of the parameter θ (associated with the variable $[\{\frac{1+z}{1-\theta'z}\} d \log (1 - ATR)]$) could be compared with the initially imposed value and an iterative procedure followed until the imposed and estimated values of θ converge; that is, until $\theta' = \theta$.

Two sets of results, obtained using the proxy variable based on the expected tax rate as described above, are shown in Table 4. The first block is for the simpler case of equation (23) where the proxy for $\Delta \log (1 - ATR)$ is used as an explanatory variable in addition to $\Delta \log (1 - \tau)$. Clearly the estimated 'income effect' is negligible and statistically insignificant, while the estimate of the elasticity of taxable income is little changed from the second part of Table 3. Using the iterative method based on the adjusted $\Delta \log (1 - ATR)$, as shown in the second part of Table 4, produces a very small negative and again insignificant income effect, θ . The coefficients on $\Delta \log (1 - \tau)$ in each case are quite stable.

Both approaches therefore suggest that income effects can indeed be neglected. The more complex case where R is not neglected has been shown to be based on the use of the Slutsky equation as a starting point, with constant parameters imposed at that initial step. It is interesting to see if the simpler expression involving $\Delta \log (1 - ATR)$ can be obtained using a different approach. This is considered in the next subsection.

²³The effective tax rate structure is not actually proportional over lower ranges because of the existence of various means-tested benefits.

²⁴One simplification is to use polynomial expansions, neglecting cubic and higher-order terms, so that $\frac{(1+z)\theta}{1-\theta z} = (1+z)\theta + z(1+z)\theta^2$ and $\frac{1}{1-\theta z} = 1 + z\theta + (0.5 + z^2)\theta^2$. However, estimation based on the resulting expression for $d \log y$ was not successful, largely due to multicollinearity problems.

Table 4: Alternative Specifications of Income Effects: Proxy Based on Expected Tax Rate

Dependent variable: $\log y_{02} - \log y_{99}$		
Independent variables	Parameter estimate	<i>t</i> -value
<i>'Simple' case</i>		
Intercept	1.70946	9.15
$\Delta \log (1 - \tau)$	0.33028	5.69
$\Delta \log (1 - ATR)$	0.02766	0.12
Age	0.03456	13.4
Age-squared	-0.00045671	-15.25
$\log y_{99}$	-0.22225	-13.20
$\log y_{99} - \log y_{98}$	-0.11458	-17.57
Other income dummy	0.03649	5.68
Adjusted $R^2 = 0.0435$; $N = 38,744$		
<i>Adjusted term in ATR</i>		
Intercept	1.60035	8.12
$\Delta \log (1 - \tau)$	0.39035	5.77
Adjusted- $\Delta \log (1 - ATR)$	-0.17712	-0.66
Age	0.0346	13.41
Age-squared	-0.000457	-15.25
$\log y_{99}$	-0.21196	-11.89
$\log y_{99} - \log y_{98}$	-0.1141	-17.48
Other income dummy	0.03678	5.73
Adjusted $R^2 = 0.0435$; $N = 38,744$		

4.2 An Alternative Derivation

The approach taken in this subsection is, instead of starting from the Slutsky equation, to specify a direct utility function. A simple form is:

$$U = \gamma c^\alpha - \theta y^\delta \tag{30}$$

where c is consumption (net income) and y is gross (taxable) income. The term, θy^δ , has a minus sign because earning gross income involves sacrifices. This is a modification of the constant marginal utility (of c) form that is known to generate a constant elasticity of taxable income specification. In this case, write the linear tax function (for the relevant tax bracket) as:²⁵

$$T(y) = \tau(y - a) \tag{31}$$

²⁵For example, see Carey *et al.* (2015, p. 56), which also includes indirect taxation in the form of a broad-based goods and services tax. The neglect of indirect taxes in studies of the elasticity of taxable income involves an – often implicit – assumption that such tax rates remain unchanged.

where a is an effective tax-free threshold. The budget constraint is:

$$c = R + y(1 - \tau) \quad (32)$$

where $R = a\tau$ is virtual income, as above. Substitute for c in U and differentiate with respect to y , giving:

$$\frac{dU}{dy} = \gamma\alpha(1 - \tau)c^{\alpha-1} - \theta\delta y^{\delta-1} \quad (33)$$

Setting $\frac{dU}{dy} = 0$ and rearranging gives:

$$1 - \tau = \left(\frac{\theta\delta}{\gamma\alpha}\right) \frac{\{R + y(1 - \tau)\}^{1-\alpha}}{y^{1-\delta}} \quad (34)$$

$$1 - \tau = \left(\frac{\theta\delta}{\gamma\alpha}\right) \left[\left(\frac{R}{y(1 - \tau)} + 1\right) y(1 - \tau)\right]^{1-\alpha} y^{\delta-1} \quad (35)$$

$$1 - \tau = \left(\frac{\theta\delta}{\gamma\alpha}\right) \left[\frac{R}{y(1 - \tau)} + 1\right]^{1-\alpha} y^{\delta-\alpha} (1 - \tau)^{1-\alpha} \quad (36)$$

when $\tau = 0$, then the corresponding taxable income, y_0 , is given by:

$$1 = \left(\frac{\theta\delta}{\gamma\alpha}\right) y_0^{\delta-\alpha} \quad (37)$$

Hence:

$$(1 - \tau)^\alpha = \left[\frac{R}{y(1 - \tau)} + 1\right]^{1-\alpha} \left(\frac{y}{y_0}\right)^{\delta-\alpha} \quad (38)$$

And:

$$\frac{y}{y_0} = (1 - \tau)^{\frac{\alpha}{\delta-\alpha}} \left[\frac{R}{y(1 - \tau)} + 1\right]^{\frac{\alpha-1}{\delta-\alpha}} \quad (39)$$

Taking logarithms gives:

$$\log y = \log y_0 + \frac{\alpha}{\delta - \alpha} \log(1 - \tau) + \frac{\alpha - 1}{\delta - \alpha} \log \left[\frac{R}{y(1 - \tau)} + 1 \right] \quad (40)$$

Consider the term in square brackets, remembering that $T(y) = \tau(y - a)$ and $R = a\tau$.

Hence:

$$\frac{R}{y(1 - \tau)} + 1 = \frac{y - \tau(y - a)}{y(1 - \tau)} \quad (41)$$

$$= \left(\frac{1}{1 - \tau}\right) \left(1 - \frac{T(y)}{y}\right) \quad (42)$$

$$= \left(\frac{1}{1 - \tau}\right) (1 - ATR) \quad (43)$$

Hence:

$$\log y = \log y_0 + \frac{\alpha}{\delta - \alpha} \log(1 - \tau) - \left(\frac{\alpha - 1}{\delta - \alpha} \right) \log(1 - \tau) + \frac{\alpha - 1}{\delta - \alpha} \log[1 - ATR] \quad (44)$$

and:

$$\log y = \log y_0 + \left(\frac{1}{\delta - \alpha} \right) \log(1 - \tau) + \left(\frac{\alpha - 1}{\delta - \alpha} \right) \log[1 - ATR] \quad (45)$$

Finally:

$$\Delta \log y = \left(\frac{1}{\delta - \alpha} \right) \Delta \log(1 - \tau) + \left(\frac{\alpha - 1}{\delta - \alpha} \right) \Delta \log[1 - ATR] \quad (46)$$

Writing this as:

$$\Delta \log y = A \Delta \log(1 - \tau) + B \Delta \log[1 - ATR] \quad (47)$$

Then values of δ and α can be recovered from A and B using $\alpha = \frac{B+A}{A}$ and $\delta = \frac{1+A+B}{A}$. If it is desired to include a simple term in $\Delta \log[1 - ATR]$, it is therefore preferable to abandon the approach that starts from the Slutsky equation and instead start from the simple utility function used here.

4.3 The Elasticity of Taxable Income

The simplification from the absence of income effects can be seen further by considering the elasticity of taxable income. With income effects, the elasticity must recognise that a change in one marginal tax rate affects all those whose income falls within or above the relevant tax bracket. Consider those in the k th tax bracket, and write:

$$1 - ATR_k = \frac{R_k + y(1 - \tau_k)}{y} \quad (48)$$

then:

$$\Delta \log[1 - ATR_k] = \Delta \log(R_k + y(1 - \tau_k)) - \Delta \log y \quad (49)$$

And substituting in (46):

$$\left(\frac{\delta - 1}{\delta - \alpha} \right) \Delta \log y = \left(\frac{1}{\delta - \alpha} \right) \Delta \log(1 - \tau_k) + \left(\frac{\alpha - 1}{\delta - \alpha} \right) \Delta \log[R_k + y(1 - \tau)] \quad (50)$$

or:

$$\Delta \log y = \left(\frac{1}{\delta - 1} \right) \Delta \log(1 - \tau_k) + \left(\frac{\alpha - 1}{\delta - 1} \right) \Delta \log[R_k + y(1 - \tau)] \quad (51)$$

and the elasticity of taxable income is:

$$\eta_{y,1-\tau_k} = \frac{1}{\delta - 1} + \left(\frac{\alpha - 1}{\delta - 1} \right) \frac{\Delta \log[R_k + y(1 - \tau_k)]}{\Delta \log(1 - \tau_k)} \quad (52)$$

In the simpler case of no income effects, $\alpha = 1$ and $\eta_{y,1-\tau_k} = \frac{1}{\delta - 1} = \beta$ for all k .

5 Income Effects using Virtual Income

An alternative approach to the investigation of income effects was used by Kleven and Schultz (2014). They used a specification, ignoring other variables, as follows:

$$d \log y = \beta [d \log (1 - \tau)] + \theta d \log R \quad (53)$$

The term, $d \log R$, may be constructed from information on the tax function using $dR_k = a_k d\tau_k - db_k$ and $db_k = \sum_{j=1}^{k-1} (a_{j+1} - a_j) d\tau_j$. That is, the pre- and post-reform tax structures may be used to construct pre- and post-reform values of R_k for each taxpayer for whom $d\tau_k$ is the change in their marginal tax rate.

Table 5: Income Effects using Virtual Income

Dependent variable: $\log y_{02} - \log y_{99}$		
Independent variables	Parameter estimate	t -value
<i>Proxy based on initial income</i>		
Intercept	1.1716	11.39
$\Delta \log (1 - \tau)$	0.3118	2.63
$\Delta \log R$	-0.00002259	-0.39
Age	0.03516	13.64
Age-squared	-0.0004634	-15.47
$\log y_{99}$	-0.17236	-19.83
$\log y_{99} - \log y_{98}$	-0.11359	-17.52
Other income dummy	0.03799	5.96
Adjusted $R^2 = 0.0428$; $N = 38,744$		
<i>Proxy based on expected tax rate</i>		
Intercept	1.73925	16.88
$\Delta \log (1 - \tau)$	0.5198	5.79
$\Delta \log R$	-0.04441	-2.35
Age	0.03451	13.38
Age-squared	-0.00045619	-15.22
$\log y_{99}$	-0.219	-25.41
$\log y_{99} - \log y_{98}$	-0.11321	-17.40
Other income dummy	0.03704	5.78
Adjusted $R^2 = 0.0436$; $N = 38,744$		

As with the need to use a proxy variable for $d \log(1 - \tau)$, a similar issue arises with dR_k in which a proxy is required for the marginal tax rate change, $d\tau_k$. Similarly, db_k is obtained using the change in inframarginal tax rates for each taxpayer, $d\tau_j$, where these depend on

the post-reform tax bracket, k , associated with post-reform taxable income, y_k . Hence, like $d\tau_k$ or $d(1 - \tau_k)$, the values of db_k will differ depending on the proxy used.

Table 5 reports regression results based on (53) using the proxy based on initial income and that based on the expected tax rate. The use of initial income as a proxy suggests insignificant income effects and a coefficient on $\Delta \log(1 - \tau)$ that is similar to results reported above. For the proxy based on the expected tax rate, the coefficient on $\Delta \log R$ is small but significant. The coefficient on $\Delta \log(1 - \tau)$ is slightly higher than results shown above, although it is necessary to recognise that the elasticity of taxable income, $\eta_{y,1-\tau_k}$, is no longer simply equal to β . This is discussed in the following subsection.

5.1 Taxable Income Elasticities

The elasticity of taxable income, using the virtual income term (53), is:

$$\eta_{y,1-\tau_k} = \beta + \theta \frac{\Delta \log R_k}{\Delta \log(1 - \tau_k)} \quad (54)$$

Or:

$$\eta_{y,1-\tau_k} = \beta + \theta \frac{\Delta R_k}{\Delta(1 - \tau_k)} \left(\frac{1 - \tau_k}{R_k} \right) \quad (55)$$

Hence the elasticity can be written as:

$$\eta_{y,1-\tau_k} = \beta + \theta \eta_{R_k,1-\tau_k}$$

which maintains symmetry for the elasticities on the right hand side and left hand side of (55). This emphasises that, even if θ is small, if $\eta_{R_k,1-\tau_k}$ is sufficiently large then the taxable income elasticity, $\eta_{y,1-\tau_k}$, can be substantially different from β .

When reform involves changes in several marginal tax rates, $\eta_{R,1-\tau}$ may be positive or negative. For example, taxpayers in higher tax brackets, for whom inframarginal rates are changed, will be subject to a combination of more than one tax-reform induced change in R . The net effect may yield $\eta_{R,1-\tau}$ greater than, or less than, zero.

6 Conclusions

This paper has examined two important problems in the estimation of the elasticity of taxable income. The first arises from the need to deal with endogeneity arising from the fact that the marginal tax rate and taxable income are jointly determined in a multi-tax structure. The approach taken in many empirical studies has been to use instrumental variable estimation, typically using an instrument based on the tax rate applying to an

unchanged taxable income. In the New Zealand context Carey *et al.* (2015) found a substantial improvement by using an instrument based on the expected tax rate, allowing for a process of income dynamics, estimated over a period when no tax changes took place. In contrast, the approach proposed in the present paper has been to use, instead of instrumental variable estimation, ordinary least squares using proxy variables. It has been shown that more consistent results can be obtained using this approach.

Secondly, the paper has considered a potential role for income effects. One approach previously adopted has involved the addition of a term involving the proportional change in the average net-of-tax rate, $\Delta \log(1 - ATR)$, in addition to the change in the marginal rate, $\Delta \log(1 - MTR)$. It has been shown that the derivation of this specification, starting from the Slutsky equation, involves an invalid assumption (that virtual income can be neglected) at a crucial step. Nevertheless, correction for this assumption leads to empirical results for New Zealand which also support the finding that income effects associated with the 2001 tax reform are negligible and statistically insignificant. In addition, the simpler specification can be derived more appropriately from a direct utility function.

Income effects have also been examined by Kleven and Schultz (2014) via the introduction of a term involving the proportional change in virtual income, $\Delta \log R$, rather than $\Delta \log(1 - ATR)$. Estimates reported here for this specification show a very small negative, but significant, coefficient on $\Delta \log R$ when a proxy based on the expected tax rate is used, but a negligible and insignificant coefficient when a proxy based on an unchanged taxable income is used. Given the robust estimates obtained from income dynamics regressions based on current and lagged relative incomes, the latter proxy is clearly inferior *a priori*. This conclusion is also supported by the empirical results in Table 5.

The results thus tentatively suggest that – in the context of the tax change examined for New Zealand – income effects can safely be ignored, but the elasticity of taxable income is somewhat lower than previously estimated by instrumental variable methods. It would be of interest to apply the approach proposed here, involving counterfactual incomes based on estimating the systematic component of income dynamics, to other tax changes and countries whenever longitudinal individual income data are available.

Appendix A: The Expected Tax Rate

This appendix, taken from Carey *et al.* (2014), explains how the expected tax rate is computed for each individual. Given a distribution of income for each individual, conditional on income in the two years preceding the tax change, it is possible to calculate an expected tax rate. As before, let $y_{j,i}$ denote individual i 's income at time j , and let μ_j denote arithmetic mean log income at time, j . Rearranging equation (2) gives:

$$\log y_{j,i} = (\mu_j - \alpha_2 \mu_{j-1} - \alpha_3 \mu_{j-2}) + \alpha_2 \log y_{j-1,i} + \alpha_3 \log y_{j-2,i} + u_{j,i} \quad (\text{A.1})$$

Taking expectations gives:

$$E(\log y_{j,i} | y_{j-1,i}, y_{j-2,i}) = (\mu_j - \alpha_2 \mu_{j-1} - \alpha_3 \mu_{j-2}) + \alpha_2 \log y_{j-1,i} + \alpha_3 \log y_{j-2,i} \quad (\text{A.2})$$

and the variance of logarithms of conditional income is:

$$V(\log y_{j,i} | y_{j-1,i}, y_{j-2,i}) = \sigma_u^2 \quad (\text{A.3})$$

As explained earlier, in the context of the tax change in New Zealand, it is necessary to obtain values relating to 2002, given incomes in 1999 and 1998. Hence, moving forward one year gives:

$$\begin{aligned} E(\log y_{j+1,i} | y_{j-1,i}, y_{j-2,i}) &= (\mu_{j+1} - \alpha_2 \mu_j - \alpha_3 \mu_{j-1}) \\ &\quad + \alpha_2 E(\log y_{j,i} | y_{j-1,i}, y_{j-2,i}) + \alpha_3 \log y_{j-1,i} \end{aligned} \quad (\text{A.4})$$

with a variance of logarithms of:

$$V(\log y_{j+1,i} | y_{j-1,i}, y_{j-2,i}) = (1 + \alpha_2^2) \sigma_u^2 \quad (\text{A.5})$$

Finally, moving a further year forward gives:

$$\begin{aligned} E(\log y_{j+2,i} | y_{j-1,i}, y_{j-2,i}) &= (\mu_{j+2} - \alpha_2 \mu_{j+1} - \alpha_3 \mu_j) \\ &\quad + \alpha_2 E(\log y_{j+1,i} | y_{j-1,i}, y_{j-2,i}) \\ &\quad + \alpha_3 E(\log y_{j,i} | y_{j-1,i}, y_{j-2,i}) \end{aligned} \quad (\text{A.6})$$

with a conditional variance of logarithms of:

$$V(\log y_{j+2,i} | y_{j-1,i}, y_{j-2,i}) = \{1 + \alpha_2^2 (1 + \alpha_2^2) + \alpha_3^2\} \sigma_u^2 \quad (\text{A.7})$$

These last two expressions can be used to give the mean and variance of log-income in 2002 conditional on income in 1999 and 1998. The variance is of course the same for each individual.

The expected tax rate for the individual in period $j+2$, given a set of tax thresholds and rates, is obtained as follows. Suppose the income tax function has rates t_k for $k = 1, \dots, K$ applying between income thresholds a_k and a_{k+1} where $a_1 = 1$ and $a_{K+1} = \infty$. First let $E(\log y_{j+2,i} | y_{j-1,i}, y_{j-2,i}) = \mu_{j+2,i}$ and $V(\log y_{j+2,i} | y_{j-1,i}, y_{j-2,i}) = \sigma_{j+2}^2$, with:

$$\xi_{j+2,k,i} = \frac{\log a_k - \mu_{j+2,i}}{\sigma_{j+2}} \quad (\text{A.8})$$

On the assumption that the u are normally distributed, log-income is normally distributed and the probability that the individual falls into the k th bracket is:

$$P_{j+2,k,i} = N(\xi_{j+2,k+1,i} | 0, 1) - N(\xi_{j+2,k,i} | 0, 1) \quad (\text{A.9})$$

where $N(h | 0, 1)$ is the area to the left of h of a standard normal distribution. Here $N(\xi_{j+2,K+1,i} | 0, 1) = 1$ and $N(\xi_{j+2,1,i} | 0, 1) = 0$. The expected tax rate for the individual, $E(\tau_{j+2,i})$ is thus:

$$E(\tau_{j+2,i}) = \sum_{k=1}^K t_k P_{j+2,k,i} \quad (\text{A.10})$$

This gives the expected tax rate, $\tau_i^* [E(\tau_{2,i})] = E(\tau_{j+2,i})$, for each individual.

Appendix B: The Inland Revenue Data

The data used in this paper are personal income information sourced from the New Zealand Inland Revenue Department's (IRD's) tax returns and employer PAYE records. The database is a stratified random sample, including 2 per cent of all wage and salary earners (which in turn includes people in receipt of taxable welfare benefits) and 10 per cent of all other individual taxpayers, such as the self-employed. The database omits individuals with no personal taxable income (unless they filed a tax return), and those whose only income was from investments with the correct amount of tax deducted at source and no requirement to file a tax return. The former group are not of interest for this study, and the latter are expected to be a fairly small group representing a very small proportion of total taxable income. The database does not include income not attributed to natural persons, for example income held in companies or trusts.

Randomness is ensured by sampling taxpayers based on the last two digits of their unique 'IRD number', which are issued broadly sequentially and not reflective of the characteristics of the specific individual. In order to ensure these are representative of the total individual taxpayer population, weights are applied to each observation in the sample according to the characteristics of the individual. For 1999, the database includes a total sample of 138,464

individual taxpayers, representing a total population of 2,800,528 taxpayers. For 2002 the sample size increases to 139,420, representing a taxpayer population of 2,962,200.

The database covers the years 1994 to 2009, and allows users to follow individuals across time by use of their IRD number. Because filing requirements have changed across time, the dataset contains a number of structural breaks. These include a break across the 1999–2002 period considered here, when the pre-populated personal tax summary (PTS) replaced the old IR5 tax return. This had a minor impact on some income tax data collected, particularly with regards to dividend and interest income below a small threshold. Aside from salary and wage income data, the database also includes data on business income, trust income, interest, dividends, rental income, shareholder-employee salary, partnership income and other income. Expenses and losses claimed (including those through LAQCs) are also recorded, as well as information on demographic characteristics such as date of birth and gender. These data are taken from a range of sources, largely tax returns submitted to the IRD.

For the regressions in this study, various restrictions are applied to the data. Firstly, in recognition that various unrelated behavioural changes may bias the results, those taxpayers who were younger than 25 in 1999, or older than 64 in 2002, are removed from the sample. This fairly common restriction removes those taxpayers likely to be in the very early stages of a career, as well as those likely to have retired at the age of 65 (the age of eligibility for New Zealand superannuation).

Secondly, those with 1999 taxable income less than \$16,000 or greater than \$1,000,000 are excluded from the sample. The first of these restrictions is particularly important in order to remove a significant segment of the population who received some form of government benefit, as abatement rates mean that these individuals face different effective marginal tax rates to standard taxpayers.

Finally, the sample is necessarily reduced to only those individuals who have sufficient data in all six relevant income years (ending 1998, 1999, 2002, 2003, 2004 and 2005). Some taxpayers either entered or exited the tax system over this time, which means that their income dynamics cannot be estimated. A number of smaller, less significant restrictions are also imposed, such as the removal of zero or negative taxable income values and data entry errors (such as negative ages). Combined, these restrictions reduce the sample size to 38,744, which, when weighted up to reflect the population, represents 803,920 individual taxpayers (29 per cent of the original 1999 weighted sample).

Appendix C: Income Dynamics Parameter Estimates

Section 3.1 discussed the income dynamic specification used to identify the regression to the mean and serial correlation parameters, ϕ and γ respectively. It also explained that, though a three-year period *after* the 2001 reform was selected to obtain the income dynamic parameters used to construct the expected tax rate proxy, regressions for other years not involving major tax reforms yield similar estimates. Figure 1 reports results obtained by Laws (2014) for a slightly different sample from that used in this paper. These nevertheless illustrate the lack of variation in parameter estimates associated with different estimation periods.

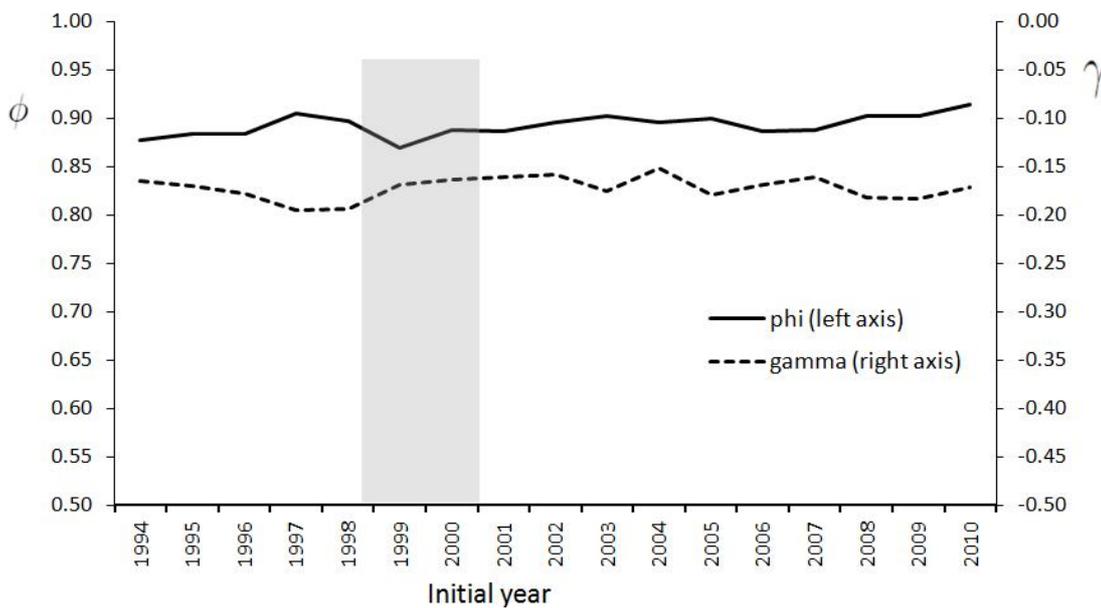


Figure 1: Income Dynamics: Parameter Estimates from Laws (2014)

In Figure 1, the year on the horizontal axis represents the first of the three years ($t - 2$) used for estimation involving incomes in years t , $t - 1$ and $t - 2$. The two years identified by the shaded area are years in which the three-year estimation period includes *both* 2000 and 2001 (that is, 1999-2001 and 2000-2002). Both parameter estimates are relatively stable across the whole period from 1994 to 2012, but with a noticeable fall in ϕ associated with the 2000-01 tax reforms. This confirms both that the parameter estimates are relatively insensitive to the specific non-reform period used for estimation of (2), and that income dynamics associated with the tax reform period are likely to have been affected by that reform, consistent with the evidence presented in subsection 4.1 and section 5 above.

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